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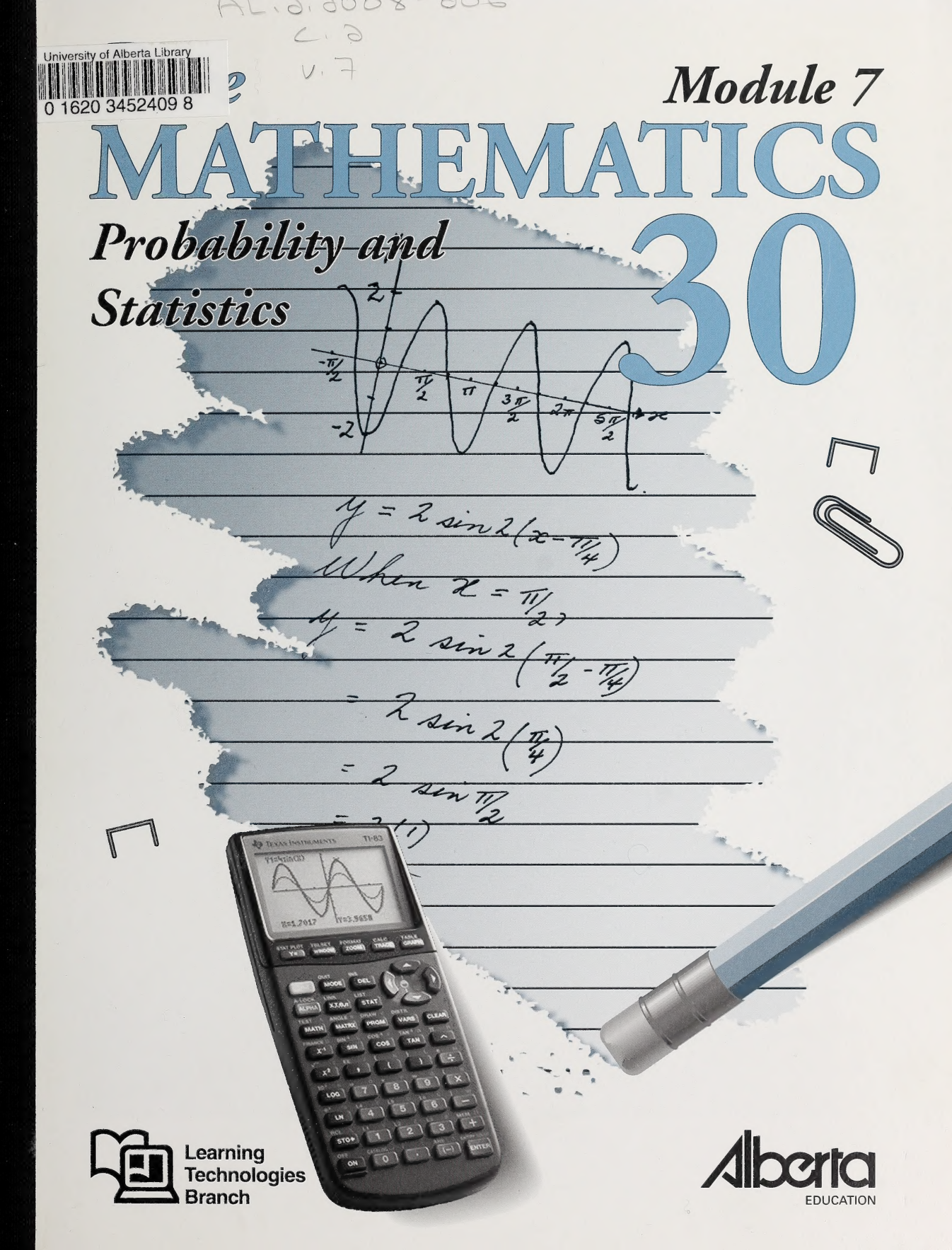
Module 7

MATHEMATICS 30

Probability and Statistics



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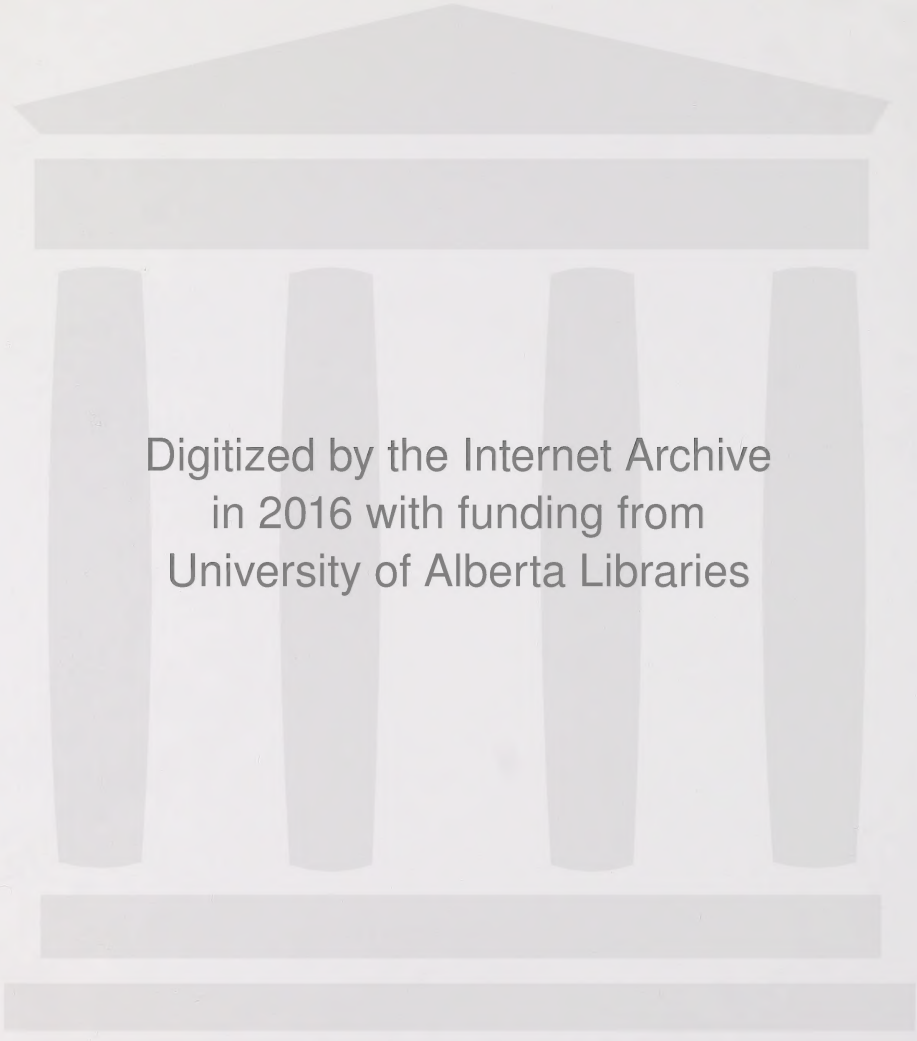
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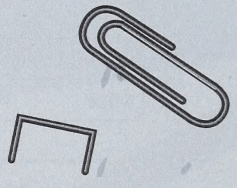
Pure MATHEMATICS 30

Module 7
Probability and Statistics



Pure Mathematics 30
Module 7: Probability and Statistics
Student Module Booklet
Learning Technologies Branch
ISBN 0-7741-2813-5

The Learning Technologies Branch acknowledges with appreciation the Alberta Distance Learning Centre and Pembina Hills Regional Division No. 7 for their review of this Student Module Booklet.

This document is intended for	
Students	✓
Teachers	✓
Administrators	
Home Instructors	
General Public	
Other	

You may find the following Internet sites useful:

- Alberta Education, **http://www.education.gov.ab.ca**
- Learning Technologies Branch, **http://www.education.gov.ab.ca/ltb**
- Learning Resources Centre, **http://www.lrc.education.gov.ab.ca**

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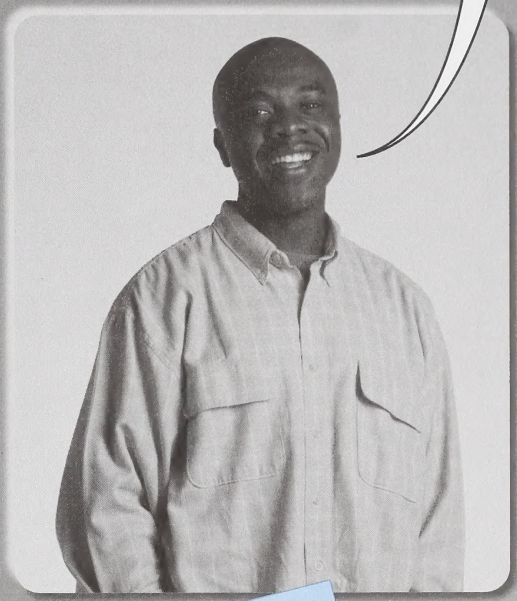
Welcome

Pure Mathematics 30

- **Module 1**
  Transformations
- **Module 2**
  Exponents and Logarithms
- **Module 3**
  Sequences and Series
- **Module 4**
  Conics
- **Module 5**
  Trigonometry
- **Module 6**
  Combinatorics
- **Module 7**
  Probability and Statistics

Welcome to **Module 7**.
We hope you'll enjoy your study of **Probability and Statistics**.



Module 1 contains general information about the course components, additional resources, icons, assessment, and strategies for completing your work. If you do not have access to Module 1, contact your teacher to obtain this important information.

It is recommended that you work through the modules in the order given (from 1 to 7), since several concepts build on each other as you progress through the course.

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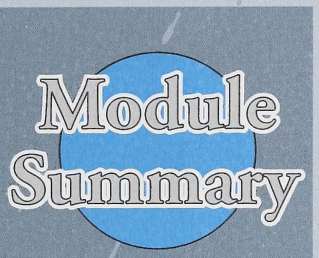

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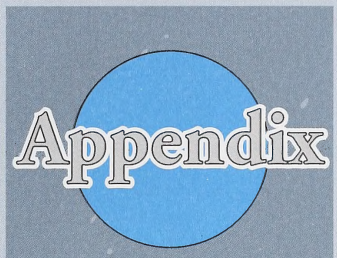

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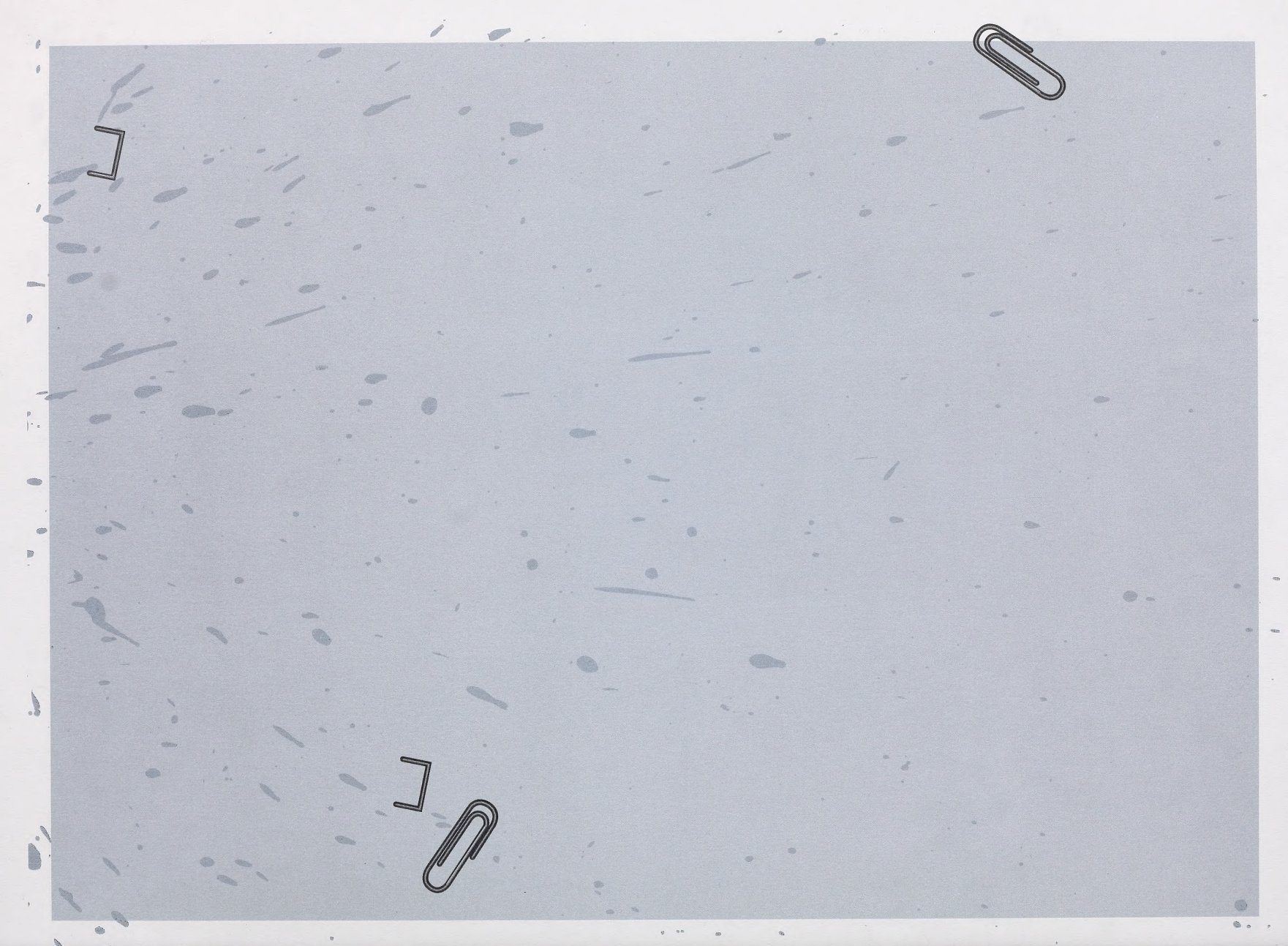


Module Overview

When you read a newspaper, what section do you turn to first? Do you start with the sports section to check the latest scores, standings, and player statistics? Do you examine the business section to see the latest figures on the economy? Perhaps you check the weather forecast to find the probability of rain or inclement weather so you can plan your day.

Regardless of which section you choose, reporters present data to persuade, inform, or entertain you. But how valid or reliable is this information? A knowledge of probability and statistics will help you interpret and critique data that is presented to you.

In this module you will determine probabilities and explore statistics. In Section 1 you will review and extend your knowledge of theoretical probability, exploring the probabilities of a variety of events. Throughout your exploration you will apply your knowledge of combinations, permutations, and the Binomial Theorem. In Section 2 you will expand your work in probability to study two probability distributions used to analyze and present statistical data.

Module 7: Probability and Statistics

Assessment

The document you are presently reading is called a Student Module Booklet. It will show you, step by step, what to do and how to do it.

This module, Probability and Statistics, has two sections. Within each section, your work is grouped into activities. Within the activities, there are readings and questions for you to answer. By completing these questions you will construct your own learning, discover mathematical connections, and practise or apply what you have learned. The suggested answers in the Appendix of this Student Module Booklet will provide you with immediate feedback on your progress.

In this module you will be directed to the accompanying Assignment Booklets. You are expected to complete two Assignment Booklets for this module. Your grading in this module is based upon the assignments that you submit for assessment. The mark distribution is as follows:

Assignment Booklet 7A	
Section 1 Assignment	**35 marks**
Assignment Booklet 7B	
Section 2 Assignment	**35 marks**
Final Module Assignment	**30 marks**
TOTAL	**100 marks**

Strategies for Completing a Module

Before you begin, organize your materials: Student Module Booklet, textbook, Assignment Booklets, binder, lined paper, graph paper, graphing calculator, pens, pencils, and so on. Make sure you have a quiet area in which to work, away from distractions. Set up your time schedule.

To achieve success in this module, be sure to read all your instructions carefully and work slowly and systematically through the material. Remember, it's the work you do in this Student Module Booklet that will prepare you for your assignments and final test. Try to set realistic goals for yourself each day; and once you've set them, stick to them. Submit your assignments regularly, and don't forget to review your work before handing it in. Careful work habits will greatly increase your chances for success in Pure Mathematics 30.

Good luck!

Section 1

ED ECKSTEIN/CORBIS

Probability

Many people have dreamed about what they would do if they won the lottery. Most people, though, realize that their chances of winning are extremely remote.

Imagine a swimming pool filled with almost 14 million red jellybeans. Hidden amongst the millions is one black jellybean. Put on a blindfold and wade through the candies; then stop, reach down, and select a single jellybean. Your chances of selecting the black jellybean are almost the same as your chances of selecting the correct 6-number combination out of 49 numbers in a popular lottery!

In this section you will explore the probabilities of familiar events. You will use probability tree diagrams, Venn diagrams, and the principles of combinatorics in your exploration. You will also classify and distinguish among a variety of different events, identify relationships among their probabilities, and apply a number of probability techniques to solve problems.

Activity 1: Basic Probability—A Review

Mathematical Process

- Communication
- [X] Connection
- Estimation
- Mental Math
- Problem Solving
- Reasoning
- Technology
- Visualization

A camping trip is planned for the weekend and Lisa asks her parents for permission to attend. What are her chances? How probable is it that her parents will give their consent? Is it 100%, 0%, or perhaps some percentage in between?

Many everyday events are expressed in terms of **probability**. What is the probability of rain tomorrow? What is the probability that the local hockey team will make the playoffs? What is the probability of winning the lottery?

probability: the study of chance and uncertainty

In earlier mathematics courses you studied experimental and theoretical probability. In this activity you will review probability and some of its basic terms and principles.

Turn to page 368 of *MATHPOWER 12* and read "Probability and Sample Space."

1. Answer the following on page 368 of the textbook.

a. question 1 of "Explore: Use a Diagram"
b. question 1 of "Inquire"

Turn to page 88 to compare your responses with those in the Appendix.

experiment: *an action that has measurable or quantifiable results*

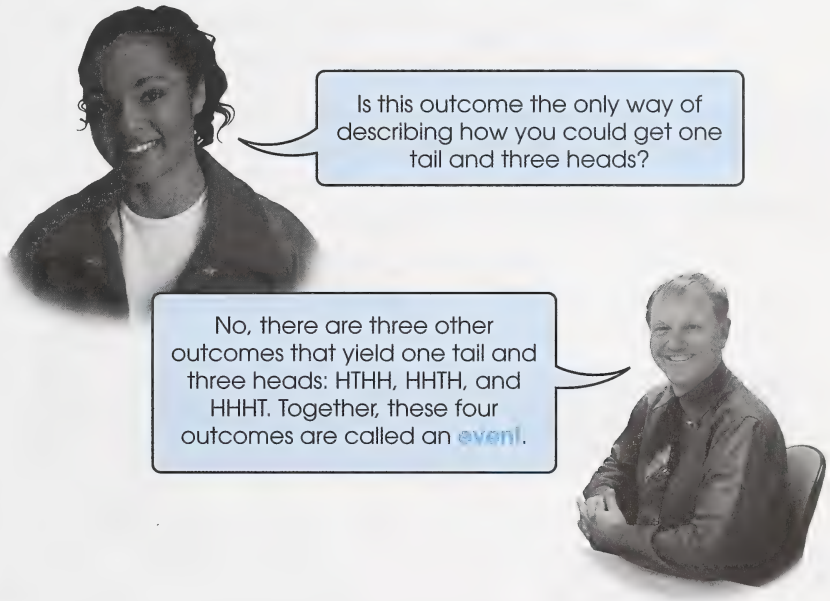
outcome: *the result of a trial of an experiment*

sample space: *the entire set of outcomes possible*

event: *a set of outcomes of an experiment*

In the preceding questions you looked at what could happen if you tossed four fair coins. The act of tossing the four coins is called an **experiment**. In probability theory, an experiment is an operation with definite results. Because each coin could turn up two ways, you discovered, by completing a tree diagram, that there were $2 \times 2 \times 2 \times 2 = 16$ possible results, or **outcomes**, that are equally likely to occur.

The entire set of outcomes possible is called the **sample space**. One of the outcomes in the sample space is THHH. This outcome is obtained when the first coin turns up tails and the second, third, and fourth coins turn up heads.

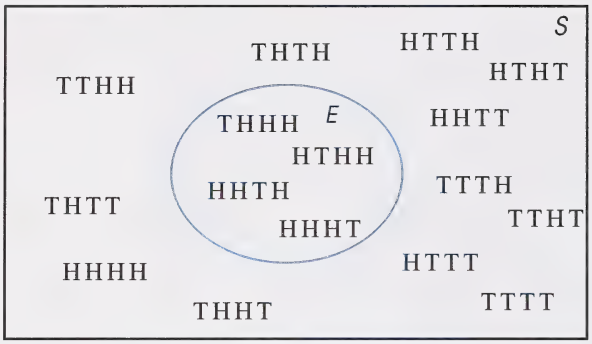

Is this outcome the only way of describing how you could get one tail and three heads?
No, there are three other outcomes that yield one tail and three heads: HTHH, HHTH, and HHHT. Together, these four outcomes are called an event.

An event is a subset of the sample space. The probability of an event for which each of the outcomes in the event is equally likely to occur (such as obtaining three heads and a tail) is given by the following formula:

$$P(\text{event}) = \frac{\text{number of outcomes favourable to the event}}{\text{total number of outcomes in the sample space}}$$

For three heads and one tail, the number of favourable outcomes is 4 and the total number of outcomes in the sample space is 16.

This may be easier to visualize using a **Venn Diagram**.



The favourable outcomes are in the circle representing the event, E. All possible outcomes are in the rectangle representing the sample space, S.

Therefore, $P(3 \text{ heads, } 1 \text{ tail}) = \frac{4}{16}$

$= \frac{1}{4}$ or 0.25 or 25%

Remember: The probability of an event can be expressed as a common fraction, a decimal, or a percent.

Example

Determine the probability of each of the following events when 4 coins are tossed.

a. obtaining 5 heads
b. obtaining either heads or tails on the first coin

Mathematical Process

- Communication
- Connection
- Estimation
- Mental Math
- Problem Solving
- Reasoning
- Technology
- ☒ Visualization

Venn diagram: a diagram that uses overlapping circles in a rectangle to illustrate the relationship among sets

Solution

a. You cannot obtain five heads when only four coins are tossed. Therefore, the number of outcomes favourable to the event is 0.

$$\therefore P(\text{event}) = \frac{\text{number of outcomes favourable to the event}}{\text{total number of outcomes in the sample space}}$$

$$P(5\text{ H}) = \frac{0}{16}$$
$$= 0 \text{ or } 0\%$$

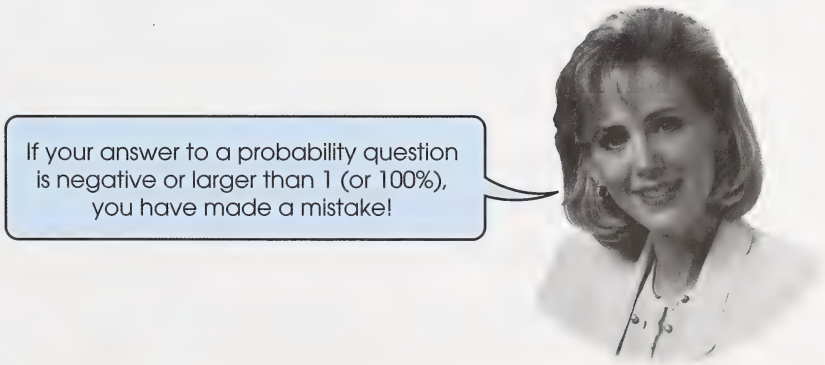
b. The first coin will always turn up either heads or tails. Therefore, the number of outcomes favourable to the event is 16.

$$\therefore P(\text{event}) = \frac{\text{number of outcomes favourable to the event}}{\text{total number of outcomes in the sample space}}$$

$$P\left(\begin{array}{c}\text{heads or tails} \\ \text{on the first coin}\end{array}\right) = \frac{16}{16}$$
$$= 1 \text{ or } 100\%$$

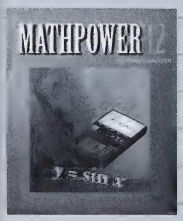
If $P(E) = 0$, event E is called an **impossible event**—an event that never occurs.
If $P(E) = 1$, event E is called a **certain event**—an event that always occurs.

Therefore, for any event, E, $0 \leq P(E) \leq 1$.



Using a tree diagram is another way of visualizing a relationship.

Turn to page 368 of *MATHPOWER 12* and read from the red line to the red line on page 370, working through Example 1.



You can use tree diagrams to obtain not only the outcomes of a sample space, but also the probability of each of the outcomes in the sample space. You can do this by assigning probabilities to the branches of the tree diagram. This type of tree diagram is called a **probability tree**.

probability tree: a tree diagram that includes probabilities

Mathematical Process
- Communication
- Connection
- Estimation
- Mental Math
- Problem Solving
- Reasoning
- Technology
- [x] Visualization

Example

A coin is tossed and a marble is selected from a bag containing two red marbles and one white marble. Use a probability tree to determine the probability of

a. tossing a head and selecting a red marble

b. tossing a tail and selecting a white marble

c. tossing a head and selecting a red marble or tossing a tail and selecting a white marble

Solution

a. Method 1: Distinguishing Between Red Marbles

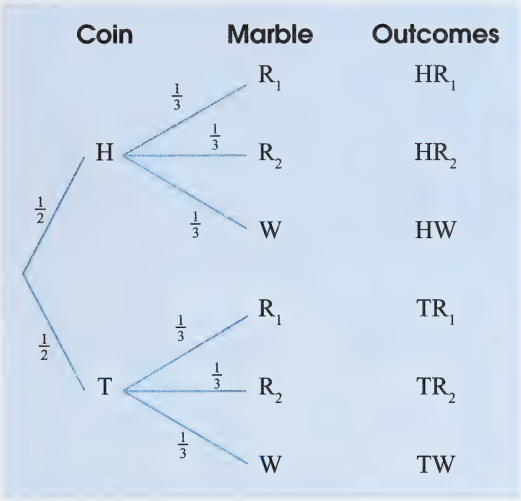
Let H represent heads, and let T represent tails.

Let R represent a red marble, where R_1 is the first red marble and R_2 is the second red marble; and let W represent the white marble.

When the coin is tossed, $P(\mathrm{H}) = \frac{1}{2}$ and $P(\mathrm{T}) = \frac{1}{2}$.

When a marble is selected, $P(\mathrm{R}_1) = \frac{1}{3}$, $P(\mathrm{R}_2) = \frac{1}{3}$, and $P(\mathrm{W}) = \frac{1}{3}$.

Draw a probability tree.



From the diagram, there are 6 possible outcomes, each equally likely to occur $\frac{1}{6}$ of the time. Notice that $\frac{1}{6}$ is the product of the probabilities on the branches.

$$\frac{1}{2}\times\frac{1}{3}=\frac{1}{6}$$

There are two outcomes of the form (H, R).

$$\therefore\ P(\text{H, R})=\frac{1}{6}+\frac{1}{6}$$
$$=\frac{1}{3}$$

Method 2: Not Distinguishing Between Red Marbles

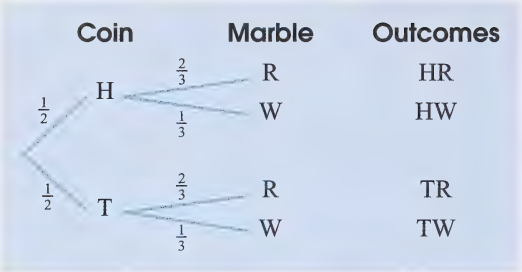
Let H represent heads; let T represent tails; let R represent a red marble; and let W represent a white marble.

When the coin is tossed, $P(\text{H})=\frac{1}{2}$ and $P(\text{T})=\frac{1}{2}$.

Because there are two red marbles and one white marble,

$P(\text{R})=\frac{2}{3}$ and $P(\text{W})=\frac{1}{3}$.

Draw a probability tree.



This answer can be found by multiplying the probabilities along the branches of the probability tree until the outcome HR is reached.

$$\therefore P(\text{H, R}) = \frac{1}{2} \times \frac{2}{3}$$
$$= \frac{1}{3}$$

b. Using either probability tree,

$$P(\text{T, W}) = \frac{1}{2} \times \frac{1}{3}$$
$$= \frac{1}{6}$$

c. $P(\text{HR or TW}) = P(\text{HR}) + P(\text{TW})$

$$= \frac{1}{3} + \frac{1}{6}$$
$$= \frac{2}{6} + \frac{1}{6}$$
$$= \frac{3}{6}$$
$$= \frac{1}{2}$$

Turn to pages 370 and 371 of *MATHPOWER 12* and work through Example 2.

2. Answer the following on pages 371 and 372 of the textbook.

a. questions 4, 5, and 6 of "Practice"

b. questions 9, 10, and 14.a. to 14.e. of "Applications and Problem Solving"

Turn to page 89 to compare your responses with those in the Appendix.

Mathematical Process

- ☐ Communication
- ☐ Connection
- ☐ Estimation
- ☐ Mental Math
- ☒ Problem Solving
- ☒ Reasoning
- ☐ Technology
- ☐ Visualization

Example

A jar contains 3 red marbles and 2 white marbles. Two marbles are selected from the jar, and their colours are recorded. Determine the probability that the second marble is red if

a. the first marble is placed back in the jar before the second marble is drawn

b. the first marble is not placed back in the jar before the second marble is drawn

Solution

a. If the first marble is placed back in the jar before the second marble is drawn, it does not matter if the first marble was red or white. Therefore, the probability of selecting a red marble on the second draw is

$$P = \frac{\text{number of red marbles}}{\text{total number of marbles}}$$
$$= \frac{3}{5}$$

What happens in the first draw does not affect the probability of drawing a red marble in the second draw. Therefore, these two events are independent.

independent event:
an event that is not affected by the occurrence of other events

b. If the first marble is not placed back in the jar before the second marble is drawn, it does matter if the first marble drawn was red or white because there are only 4 marbles remaining in the jar for the second draw.

If the first marble was white, the probability of choosing a red marble is

$$P = \frac{\text{number of red marbles}}{\text{total number of marbles}}$$
$$= \frac{3}{4}$$

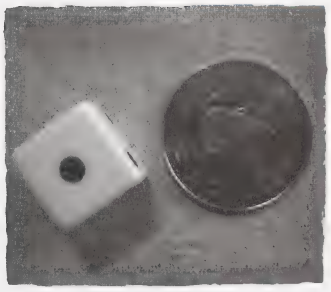
If the first marble was red, the probability of choosing a red marble is

$$P = \frac{\text{number of red marbles}}{\text{total number of marbles}}$$
$$= \frac{2}{4}$$
$$= \frac{1}{2}$$

Here, what happens in the first draw does affect the probability of drawing a red marble in the second draw. Therefore, these two events are dependent.

dependent event:
an event that is affected by the occurrence of other events

If two events are independent, there is a simple rule for determining the probability that both events will occur.



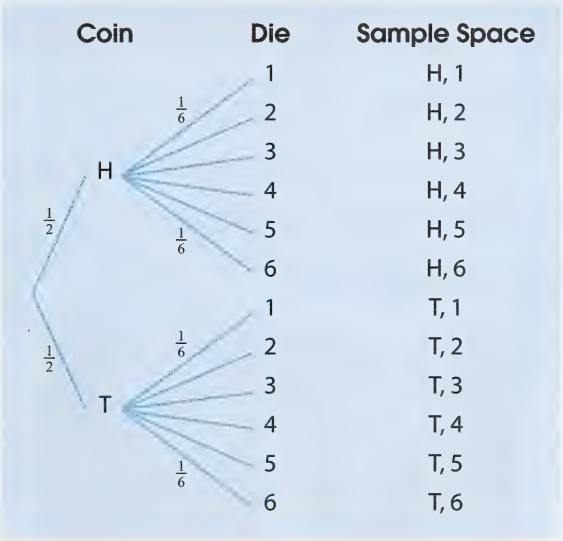
Example

A coin is tossed and a die is rolled. What is the probability that the coin turns up tails and the die shows a 1?

Solution

The events are independent. The outcome of the coin toss does not affect the roll of the die.

Draw a probability tree diagram.



As in Activity 1, multiply the probabilities on the branches of the probability tree that lead to the desired outcome.

$$P(\text{T and } 1) = P(\text{T}) \times P(1)$$
$$= \frac{1}{2} \times \frac{1}{6}$$
$$= \frac{1}{12}$$

Occasionally, when you are investigating the probability of an event, it is easier to determine the probability that the event will not occur.

Example

A multiple-choice question on a Social Studies exam has 4 choices. Eric does not know the answer; so, he is going to guess.

a. What is the probability that Eric will choose the right answer?
b. What is the probability that Eric will not choose the right answer?
c. What is the relationship between the answers to a. and b.?

Solution

a. There is one right answer and four choices. Therefore, the probability that Eric will choose the right answer is $\frac{1}{4}$.

b. There are three wrong answers and four choices. The probability that Eric will not choose the right answer is $\frac{3}{4}$.

c. $P(\text{Right}) + P(\text{Wrong}) = \frac{1}{4} + \frac{3}{4}$
$= 1$

The sum of the probabilities in questions a. and b. is 1.

complementary event: written $\overline{A}$, an event consisting of all outcomes in the sample space not in event A

The preceding example illustrates **complementary events**. If $P(A)$ is the probability that event A will occur, then $P(\overline{A})$ is the probability that event A will not occur.

Events A and $\overline{A}$ are called complementary events, and the symbol $\overline{A}$ is read "not A."

For complementary events,

$$P(A) + P(\overline{A}) = 1 \text{ or } P(A) = 1 - P(\overline{A})$$

Example

State the complement of each of the following events.

a. rolling a die and obtaining a number larger than 1
b. having 5 children and at least one child is a girl

Solution

a. rolling a die and obtaining a 1
b. having 5 children and they are all boys

Example

A card is drawn at random from a standard deck. The card is recorded and placed back in the deck. If there are a total of 6 draws, determine the following. Round your answers to 2 decimal places.

a. What is the probability that all the cards are different?
b. What is the probability that at least two are the same?

Solution

a. The probability that the second card is different from the first is $\frac{51}{52}$.

The probability that the third card is different from the first two is $\frac{50}{52}$.

The probability that the fourth card is different from the first three is $\frac{49}{52}$.

The probability that the fifth card is different from the first four is $\frac{48}{52}$.

The probability that the sixth card is different from the first five is $\frac{47}{52}$.

$$\therefore P(\text{all different}) = \frac{51}{52} \times \frac{50}{52} \times \frac{49}{52} \times \frac{48}{52} \times \frac{47}{52}$$

$$= \frac{51 \times 50 \times 49 \times 48 \times 47}{52^5}$$

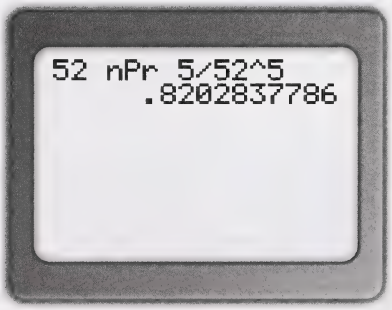
$$= \frac{{}_{51}P_5}{52^5}$$

Recall:

$${}_{51}P_5 = 51 \times 50 \times 49 \times 48 \times 47$$

To evaluate this expression using your graphing calculator, press the following.

5 2 MATH [Select the PRB menu.] 2 (2:nPr)
5 ÷ 5 2 ^ 5 ENTER



The probability that all the cards are different is approximately 0.82.

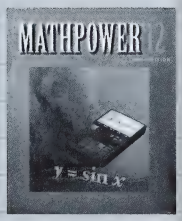
Mathematical Process
- Communication
- Connection
- Estimation
- Mental Math
- [x] Problem Solving
- Reasoning
- Technology
- Visualization

Mathematical Process
- Communication
- Connection
- Estimation
- Mental Math
- Problem Solving
- Reasoning
- [x] Technology
- Visualization

b. The complement of "all are different" is "at least two are the same."

$$\therefore P(\text{at least two are the same}) = 1 - P(\text{all different})$$
$$\doteq 1 - 0.82$$
$$\doteq 0.18$$

mutually exclusive events: ***events that cannot occur at the same time***



Turn to pages 376 to 378 of *MATHPOWER 12* and work through Example 2.

3. Answer questions 14, 15, 16.b., 16.c., 16.d., and 17 of "Applications and Problem Solving" on pages 380 and 381 of the textbook.

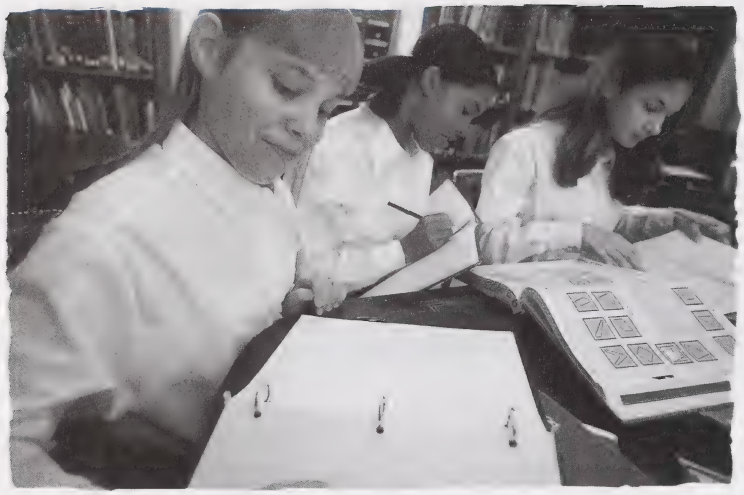
Turn to page 94 to compare your responses with those in the Appendix.

In Example 2 on pages 376 to 378 of the textbook, you worked through the answer to the question "How many students would you have to ask to have a fifty-fifty chance that two of them have the same birthday?" You may be interested in a related question: "How many students would you have to ask to have a fifty-fifty chance that one of them has the same birthday as you do?" Of course, you would have to ask more, but how many more?

If you have access to the Internet, you can generate answers to these and related birthday questions by acsessing *The Birthday Problem Applet* at the following website:

http://www-stat.stanford.edu/~susan/surprise

Now Try This



In this activity you explored the relationships among events and among the probabilities of these events. You approached your exploration systematically, using your powers of logic. The following problem will challenge your powers of logic. You may have to resort to trial and error; but even errors may help you find the answer!

4. Turn to page 381 of *MATHPOWER 12* and answer "NUMBER POWER."

Turn to page 101 to compare your response with the one in the Appendix.

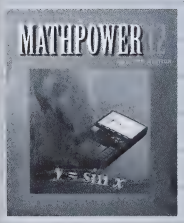
Looking Back

In this activity you reviewed the basic concepts and terminology of probability. These concepts included experiment, sample space, outcome, and event. You also defined dependent, independent, and complementary events. You then used tree diagrams to list outcomes in a sample space and used probability tree diagrams to calculate simple probabilities.

In your journal, start a vocabulary list for the new terms you encountered in this section. Beside each term, write its definition in your own words.

Mathematical Process

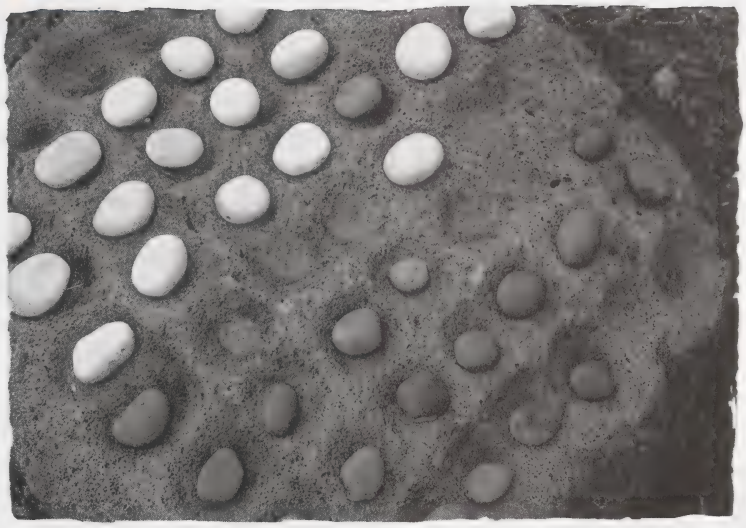
- Communication
- Connection
- Estimation
- Mental Math
- Problem Solving
- [x] Reasoning
- Technology
- Visualization



Mathematical Process

- [x] Communication
- Connection
- Estimation
- Mental Math
- Problem Solving
- Reasoning
- Technology
- Visualization

Activity 2: Probability and Combinatorics



JAMES L. AMOS/CORBIS

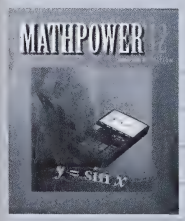
Mathematical Process

- ☐ Communication
- ☒ Connection
- ☐ Estimation
- ☐ Mental Math
- ☐ Problem Solving
- ☐ Reasoning
- ☐ Technology
- ☐ Visualization

Over the years, people have enjoyed the diversion and competition of board games. Even ancient societies have been known to play some sort of board game. (The photograph shows an ancient Hawaiian board game, similar to checkers, played with black and white stones on a grooved, stone surface.) Did you know that archaeologists have records of dice predating 700 BC?

Are you one of those people who seem to have more success at board games than others? Success is not just a matter of luck; it depends on skills gained through experience in playing the game, such as strategy and an intuitive appreciation of probability—the probability of rolling a certain sum on the dice or drawing a particular card.

In this activity you will enhance your ability to determine probabilities. You will add procedures involving the Fundamental Counting Principle, permutations, and combinations to your probability tool kit. Before you proceed, you may wish to review Module 6: Combinatorics.

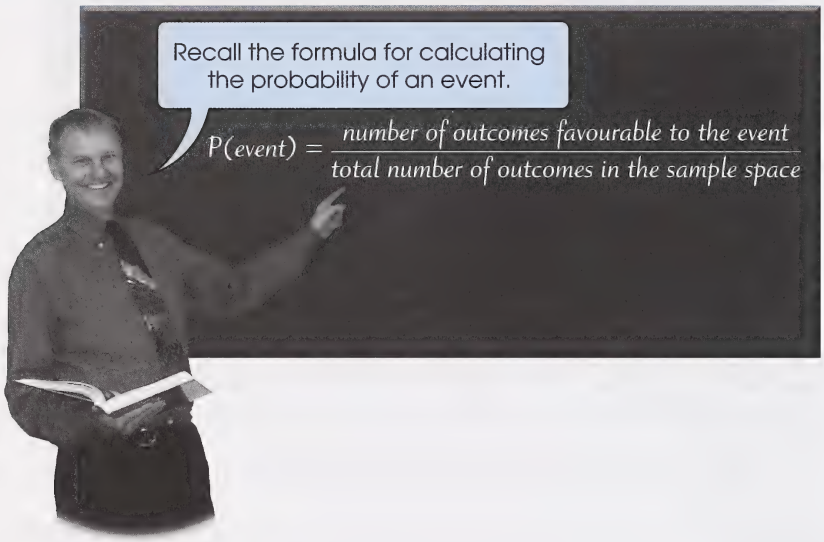


Turn to page 382 of *MATHPOWER 12* and read "Probability and Combinatorics."

1. Answer the following on page 382 of the textbook.

a. question "Explore: Draw a Tree Diagram"
b. questions 1 to 3 of "Inquire"

Turn to page 101 to compare your responses with those in the Appendix.


Recall the formula for calculating the probability of an event.
P(event) = number of outcomes favourable to the event / total number of outcomes in the sample space

The number of outcomes favourable to an event, E, is often abbreviated $n(E)$. Similarly, the number of outcomes favourable to the sample space, S, is abbreviated $n(S)$.

Therefore, the formula for the probability of an event, E, may be written as follows:

$$P(E) = \frac{n(E)}{n(S)}$$

So, to determine a probability, you must first find $n(E)$ and $n(S)$. You will find these values throughout this activity using combinatorics. Usually, you will use permutations if order matters, combinations if order is irrelevant, and the Fundamental Counting Principle to tie related permutations or combinations.

Mathematical Process
- Communication
- Connection
- Estimation
- Mental Math
- [x] Problem Solving
- Reasoning
- Technology
- Visualization

Example

A jar contains 25 nickels and 75 dimes. If four coins are chosen at random, what is the probability that the four coins total 25¢? Round your answer to 3 decimal places.

Solution

If the four coins total 25¢, there must be 3 nickels and 1 dime.

Because the order in which the coins are selected does not matter, use combinations.

The number of ways 3 nickels can be selected from 25 nickels is ${}_{25}C_3$; and the number of ways 1 dime can be selected from 75 dimes is ${}_{75}C_1$.

Using the Fundamental Counting Principle, the number of ways of selecting 3 nickels and 1 dime is $n(E) = {}_{25}C_3 \times {}_{75}C_1$.

The number of outcomes in the sample space is the number of ways 4 coins can be selected from the 100 coins in the jar. Therefore, $n(S) = {}_{100}C_4$.

$$\therefore P(E) = \frac{n(E)}{n(S)}$$
$$= \frac{{}_{25}C_3 \times {}_{75}C_1}{{}_{100}C_4}$$
$$= \frac{2300 \times 75}{3\,921\,225}$$
$$\doteq 0.044 \text{ or } 4.4\%$$

The probability that the four coins total 25¢ is approximately 0.044 or 4.4%.

Example

A newly married couple plan to have 5 children. What is the probability that they will have 3 girls and 2 boys?

Solution

The number of ways of having three girls (G) and two boys (B) is the number of distinguishable arrangements of G G G B B.

Therefore, the number of outcomes in the event is

$$\begin{aligned} n(E) &= \frac{5!}{3!2!} \\ &= 10 \end{aligned}$$

Each of five children can be a boy or a girl. Using the Fundamental Counting Principle, the number of outcomes in the sample space is

$$\begin{aligned} n(S) &= 2 \times 2 \times 2 \times 2 \times 2 \\ &= 2^5 \\ &= 32 \end{aligned}$$

$$\begin{aligned} \therefore P(E) &= \frac{n(E)}{n(S)} \\ &= \frac{10}{32} \\ &= \frac{5}{16} \end{aligned}$$

The probability that they will have three girls and two boys is $\frac{5}{16}$.

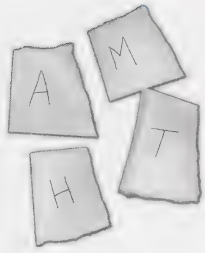
Mathematical Process

- Communication
- Connection
- Estimation
- Mental Math
- [x] Problem Solving
- Reasoning
- Technology
- Visualization

Mathematical Process

- Communication
- Connection
- Estimation
- Mental Math
- [x] Problem Solving
- Reasoning
- Technology
- Visualization

Example



The letters M, H, T, and A are written on slips of paper. The four slips of paper are placed in a hat. The slips are then selected one at a time from the hat. What is the probability that the order in which they are chosen spells MATH?

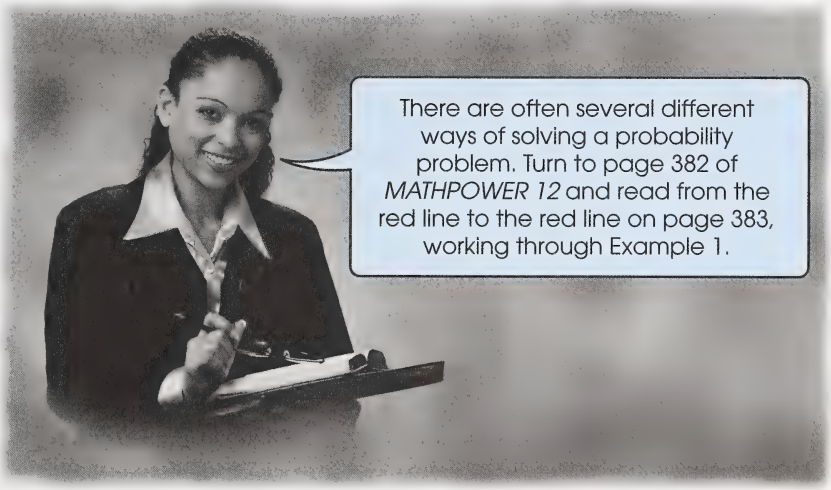
Solution

There is only one way to spell MATH; so, $n(E) = 1$.

There are 4! arrangements of the 4 letters; so, $n(S) = 4!$

$$\therefore P(E) = \frac{n(E)}{n(S)}$$
$$= \frac{1}{4!}$$
$$= \frac{1}{24}$$

The probability that the slips spell MATH is $\frac{1}{24}$.


There are often several different ways of solving a probability problem. Turn to page 382 of *MATHPOWER 12* and read from the red line to the red line on page 383, working through Example 1.

Consider the following alternate solution to Example 1.

Since Western Dancer just has to be in the top three horses, the order of finish does not matter.

If Western Dancer is in the top three, there are two other horses in the same grouping. Therefore, the number of outcomes in this event is the number of ways these two horses can be selected from the other eight horses in the race. So, $n(E) = {}_8C_2$.

The number of outcomes in the sample space is the number of ways of selecting the top three horses from the entire field of nine horses. So, $n(S) = {}_9C_3$.

$$\therefore P(E) = \frac{n(E)}{n(S)}$$
$$= \frac{{}_8C_2}{{}_9C_3}$$
$$= \frac{28}{84}$$
$$= \frac{1}{3}$$

The probability that Western Dancer finishes in the top three is $\frac{1}{3}$ or approximately 33%.

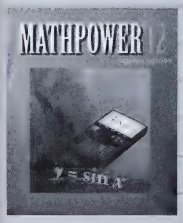
Turn to pages 383 to 385 of *MATHPOWER 12* and work through Examples 2 and 3.

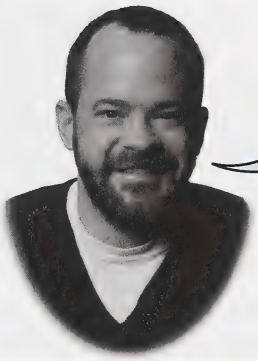
2. Answer questions 1, 2, 4, and 5 of "Practice" on page 385 of the textbook.

Turn to page 102 to compare your responses with those in the Appendix.

Mathematical Process

- Communication
- Connection
- Estimation
- Mental Math
- [x] Problem Solving
- [x] Reasoning
- Technology
- Visualization





Probability questions involving combinatorics can be complicated. You must practise to become proficient!

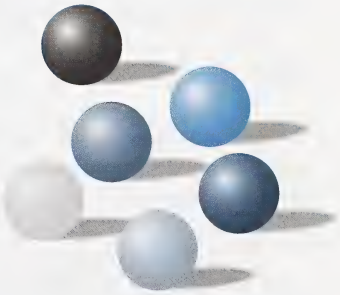
3. Turn to page 386 of *MATHPOWER 12* and answer questions 11, 12, and 14 of "Applications and Problem Solving."

Turn to page 103 to compare your responses with those in the Appendix.

Mathematical Process

- Communication
- [x] Connection
- Estimation
- Mental Math
- [x] Problem Solving
- Reasoning
- Technology
- Visualization

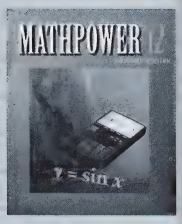
Now Try This



You know that it is possible to calculate the probability of picking a certain marble from a bag as long as you know how many marbles there are of each colour and how many marbles there are altogether. In the next question you will work backward from the probabilities to determine how many marbles there are of each colour.

4. Turn to page 386 of *MATHPOWER 12* and answer "LOGIC POWER."

Turn to page 112 to compare your response with the one in the Appendix.



Mathematical Process

- [x] Communication
- Connection
- Estimation
- Mental Math
- Problem Solving
- Reasoning
- Technology
- Visualization

Looking Back

In this activity you used your knowledge of combinatorics to answer probability questions.

In your journal, summarize the procedures you examined in this activity. Be sure to include questions you had trouble with. Write detailed solutions to these questions to help you prepare for the diploma examination.

Follow-up Activities

If you had difficulties understanding the concepts and skills in the activities, it is recommended that you do the Extra Help. If you have a clear understanding of the concepts and skills, it is recommended that you do the Enrichment. You may decide to do both.

Extra Help

One of the keys to success in solving a probability question is to read the question carefully and sort through the details in the question before you decide on a strategy. Venn diagrams or tree diagrams may help your visualize the problem, and skills you developed in Module 6 may help determine the number of outcomes. Practice is important because, in general probability, most questions fall into specific problem types.

Work through one more example before answering more questions.

Example

A CD player, a knapsack, and a basketball have been donated to a school for door prizes at an upcoming dance. There are 500 people attending the dance and everyone places their ticket stub into the draw box.

a. What is the probability of winning the CD player? What assumptions did you make?

b. Assuming the tickets are not replaced after a draw is made, what is the probability of winning any one of the remaining prizes?

c. If you are standing with nine of your friends when the prizes are drawn, what is the probability that the only prize won within your group of friends, including yourself, is the basketball? Assume the basketball is drawn last.

Solution

a. Method 1

Assuming the CD player is drawn first, the probability of winning it is $\frac{1}{500}$ because there is only 1 winning ticket chosen out of a total of 500 tickets.

Method 2

This is a question that involves combinations.

There is only one way to choose the winning ticket. The number of ways one ticket can be chosen is ${}_{500}C_1$.

$$\therefore P(\text{winning the CD player for the first prize}) = \frac{1}{{}_{500}C_1}$$
$$= \frac{1}{500}$$

The probability of winning the CD player is $\frac{1}{500}$.

b. Method 1

The probability of winning any one of the prizes is the probability of winning the first prize, winning the second prize, or winning the third prize. After each draw, the number of tickets remaining decreases by 1.

$$\therefore P(\text{winning one of 3 prizes})$$
$$= P(\text{winning the first prize}) + P(\text{winning the second prize}) + P(\text{winning the third prize})$$
$$= \frac{1}{500} + \left(\frac{\cancel{499}}{500} \times \frac{1}{\cancel{499}}\right) + \left(\frac{\cancel{499}}{500} \times \frac{\cancel{498}}{\cancel{499}} \times \frac{1}{\cancel{498}}\right)$$
$$= \frac{1}{500} + \frac{1}{500} + \frac{1}{500}$$
$$= \frac{3}{500}$$

The probability of winning any one of the three prizes is $\frac{3}{500}$.

Method 2

The number of ways to choose three winning tickets is ${}_{500}C_3$. There are ${}_{499}C_2$ ways to choose the other two winners and ${}_1C_1$ ways to be one of the winners.

$$\therefore P(\text{winning 1 of the 3 prizes}) = \frac{{}_{499}C_2 \times {}_1C_1}{{}_{500}C_3}$$

$$= \frac{\frac{499!}{497!2!}}{\frac{500!}{497!3!}}$$

$$= \frac{499!}{497!2!} \times \frac{497!3!}{500!}$$

$$= \frac{3}{500}$$

The probability of winning any one of the three prizes is $\frac{3}{500}$.

c. There is a restriction in this question because you have to make sure none of the ten people in your group wins either the first prize or the second prize. Since the order in which the prizes are given is taken into account, it is a permutation.

There will be 490 people available to win the first and second prizes. The number of people that need to be arranged is 2. So, the number of ways to award the first two winners is ${}_{499}P_2$.

Because there are ten people to pick from for the third prize, there are 10 ways to choose someone for the third prize.

Method 1

$P(\text{1 of the group of 10 wins the third prize only})$

$$= \frac{\text{number of ways 3 prizes are won, given 1 of 10 wins the 3rd prize}}{\text{total number of ways the prizes can be awarded}}$$

$$= \frac{{}_{490}P_2 \times 10}{{}_{500}P_3}$$

$$= \frac{239\,610 \times 10}{124\,251\,000}$$

$$\approx 0.019$$

Method 2

The probability of not winning the first prize and not winning the second prize and winning the third prize is

$$P = \frac{490}{500} \times \frac{489}{499} \times \frac{10}{498}$$
$$= \frac{2\,396\,100}{124\,251\,000}$$
$$\approx 0.019$$

The probability that one of the group of ten wins only the basketball is approximately 1.9%.

Try some more questions. Remember to try and establish if order matters. Make sure your answers are between 0 and 1 inclusive!

Turn to pages 396 to 398 of *MATHPOWER 12* and answer the following.

1. questions 3.b., 3.c., 4, 7, and 12.c. of "Review"

2. question 6 of "Chapter Check"

Turn to page 114 to compare your responses with those in the Appendix.

Enrichment

Suppose there are eight socks in a drawer: four black and four white. What is the least number of socks that must be drawn at random to guarantee a matching pair? To solve this problem, you can use what is called the Pigeonhole Principle.

The Pigeonhole Principle is a simple, but powerful, principle. It states that if $k + 1$ objects are put into k boxes, then at least one box contains two or more objects.

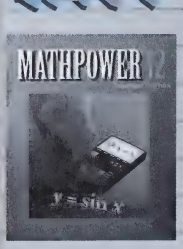
In the sock problem, the colours of the socks are the boxes. So, there are two boxes, and a pair can be guaranteed if $2 + 1 = 3$ socks are pulled from the drawer. Notice that this principle does not state which colour will be matched after three socks are pulled from the drawer, only that there is a pair.

A German mathematician by the name of Lejeune Dirichlet is credited for the Pigeonhole Principle. For more information about Dirichlet, visit the following website:

http://www-history.mcs.st-and.ac.uk/Biographies/Dirichlet.html

Turn to page 397 of *MATHPOWER 12* and read "Exploring Math: The Pigeonhole Principle"; then answer questions 1 and 2.

Turn to page 117 to compare your responses with those in the Appendix.



Conclusion

In this section you explored the probabilities of familiar events. You used probability tree diagrams, Venn diagrams, and the principles of combinatorics in your exploration. You also identified events as dependent or independent, mutually exclusive or not mutually exclusive, and complementary. You then used these classifications to help you determine appropriate probability theorems to solve a variety of practical problems.

Probability theory is not just used in games of chance, such as the lottery or the horse races. It has many more prosaic uses. Probability is a practical tool used by scientists, engineers, and others to analyze data and make decisions. Chances are you, too, will use probability at some point in your career!

Assignment

Turn to Assignment Booklet 7A and complete the assignment for Section 1. Remember to submit your completed Assignment Booklet for assessment before starting Section 2.

Section 2

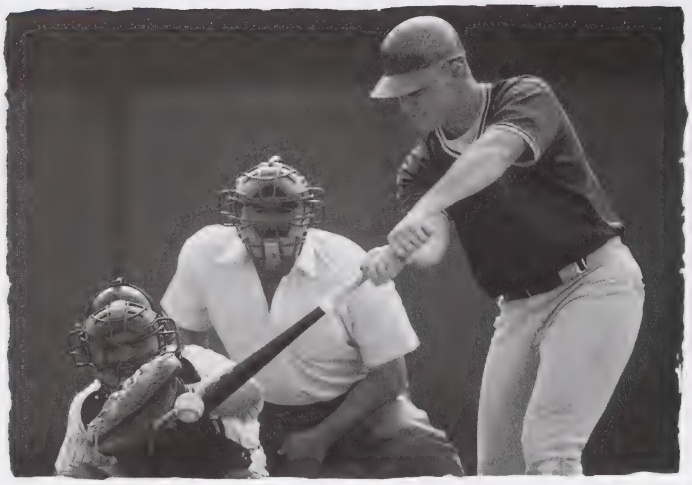
Statistics

A large appliance company determines that in order to improve its sales of a particular electric range it manufactures, the company must extend its warranty. What factors must the manufacturer take into account, particularly if the decision it makes must not adversely affect its profits? The company does not want to repair or replace too many of its ranges. Provided the manufacturer has good statistics on the durability of its range and how long the average range works before it needs repairs, the company can make the appropriate decision.

In today's world, collection and interpretation of data are becoming increasingly important. People in all fields—business, science and research, sports, and entertainment to name a few—use statistical information daily. Statistics is the branch of mathematics dealing with the collection, analysis, interpretation, and presentation of data.

In this section you will investigate data sets. You will explore data distributions using statistical measures for averages and dispersion, starting with probability distributions arising from experiments and then generalizing these distributions. You will then use the generalized distributions to analyze data, analyze probability experiments, and present data. Finally, you will look at ways of expressing the degree of confidence you have in the results you obtained through your analysis.

Activity 1: The Binomial Distribution



Mathematical Process

- Communication
- [x] Connection
- Estimation
- Mental Math
- Problem Solving
- Reasoning
- Technology
- Visualization

What is the probability of a baseball player with a batting average of 0.400 getting exactly one hit in three times at bat? What is the probability of a hockey team with a 0.500 record winning three of their next four games in the playoffs against a team with an identical record? What is the probability of getting 75% on a multiple-choice exam with 20 questions by guessing alone?

These questions are all examples of probabilities involving independent events with two possible outcomes: the baseball player will or will not get a hit each time at bat, the hockey team will either win or lose each playoff game, and the answer to each multiple-choice question is either right or wrong.

In this activity you will explore the probabilities arising from experiments (such as those described here) consisting of a series of trials that are identical, independent, and have only two possible outcomes. These experiments (or trials) are called **Bernoulli trials** (**or binomial trials**). The associated probabilities are related to the general term of the Binomial Theorem. As a result, the distributions of probabilities arising from Bernoulli trials or experiments are termed **binomial distributions**.

Bernoulli trial: an event that has only two possible outcomes: success and failure

binomial distribution: a distribution of probabilities for Bernoulli trials

Bernoulli trials were named after Jacob Bernoulli (1654–1705), one of more than a dozen mathematicians from the same family. You can find out more about Jacob Bernoulli and the Bernoulli family of mathematicians at the following website:

http://www-history.mcs.st-and.ac.uk/Biographies/Bernoulli_Jacob.html

Turn to page 404 of *MATHPOWER 12* and read "The Binomial Distribution."

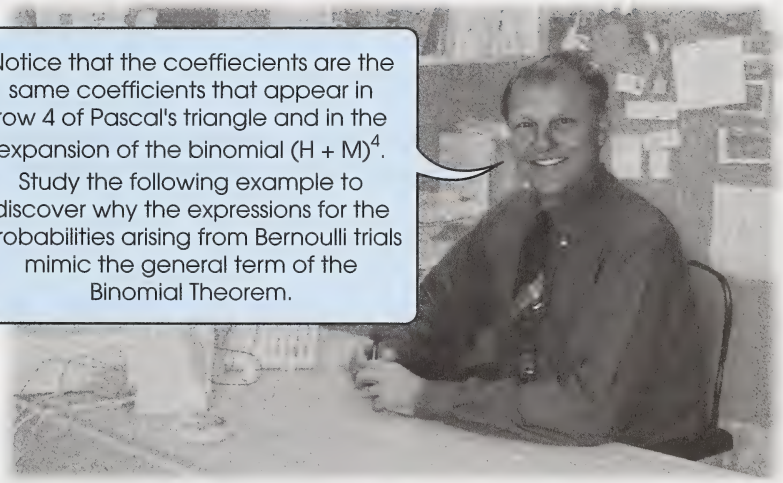
1. Answer the following on page 404 of the textbook.

a. questions a. to d. of "Explore: Look for a Pattern"
b. questions 1 to 5 of "Inquire"

Turn to page 118 to compare your responses with those in the Appendix.

When you worked through "Explore: Look for a Pattern" on page 404 of the textbook, you obtained the following distribution of probabilities for the number of hits when four arrows are shot. **Remember:** On each shot, the probability Kaia would hit the target was 0.75 and the probability she would miss the target was 0.25.

Number of Hits	Probability	Coefficient
0	${}_4C_0\,(0.75)^0\,(0.25)^4$	${}_4C_0$
1	${}_4C_1\,(0.75)^1\,(0.25)^3$	${}_4C_1$
2	${}_4C_2\,(0.75)^2\,(0.25)^2$	${}_4C_2$
3	${}_4C_3\,(0.75)^3\,(0.25)^1$	${}_4C_3$
4	${}_4C_4\,(0.75)^4\,(0.25)^0$	${}_4C_4$



Notice that the coeffiecients are the same coefficients that appear in row 4 of Pascal's triangle and in the expansion of the binomial $(H + M)^4$. Study the following example to discover why the expressions for the probabilities arising from Bernoulli trials mimic the general term of the Binomial Theorem.

Example

The multiple-choice section of a test consists of 5 questions. Each question has 4 choices and is answered by guessing.

a. What is the probability of answering the first 3 questions correctly and the last 2 questions incorrectly?

b. What is the probability of answering any 3 questions correctly?

c. Describe all the possible outcomes when the 5 questions are answered by guessing. What is the probability of each of these outcomes?

Solution

a. The probability of answering a given question correctly, $P(\text{R})$, is 0.25 because there is only 1 correct answer out of a total of 4 choices. The probability of getting the answer wrong, $P(\text{W})$, is

$$\begin{aligned} P(\text{W}) &= 1 - P(\text{R}) \\ &= 1 - 0.25 \\ &= 0.75 \end{aligned}$$

Therefore, using the Fundamental Counting Principle, the probability of answering the first three questions correctly and the last two questions incorrectly is

$$\begin{aligned} P &= (0.25)(0.25)(0.25)(0.75)(0.75) \\ &= (0.25)^3 (0.75)^2 \\ &\doteq 0.0088 \text{ or } 0.88\% \end{aligned}$$

b. The number of ways of answering any three questions correctly and two questions incorrectly is the number of distinguishable arrangements of R R R W W.

$$\begin{aligned} \text{Number of distinguishable arrangements} &= \frac{5!}{3!2!} \\ &= \frac{5!}{3!(5-3)!} \\ &= {}_5C_3 \end{aligned}$$

Because there are ${}_5C_3 = 10$ situations that lead to three correct answers, the probability of answering any three questions correctly is

$$\begin{aligned} P(\text{3 right and 2 wrong}) &= {}_5C_3\,[P(\text{R})]^3\,[P(\text{W})]^2 \\ &= 10(0.25)^3\,(0.75)^2 \\ &\doteq 0.088 \text{ or } 8.8\% \end{aligned}$$

c. When guessing on the test, you could get 0, 1, 2, 3, 4, or 5 questions correct.

The probability of each of these outcomes is as follows:

$$\begin{aligned} P(\text{0R and 5W}) &= {}_5C_0\,[P(\text{R})]^0\,[P(\text{W})]^5 \\ &= 1(0.25)^0\,(0.75)^5 \\ &\doteq 0.237 \text{ or } 23.7\% \end{aligned}$$

$$\begin{aligned} P(\text{1R and 4W}) &= {}_5C_1\,[P(\text{R})]^1\,[P(\text{W})]^4 \\ &= 5(0.25)^1\,(0.75)^4 \\ &\doteq 0.396 \text{ or } 39.6\% \end{aligned}$$

$$\begin{aligned} P(\text{2R and 3W}) &= {}_5C_2\,[P(\text{R})]^2\,[P(\text{W})]^3 \\ &= 10(0.25)^2\,(0.75)^3 \\ &\doteq 0.264 \text{ or } 26.4\% \end{aligned}$$

$$\begin{aligned} P(\text{3R and 2W}) &= {}_5C_3\,[P(\text{R})]^3\,[P(\text{W})]^2 \\ &= 10(0.25)^3\,(0.75)^2 \\ &\doteq 0.088 \text{ or } 8.8\% \end{aligned}$$

$$\begin{aligned} P(\text{4R and 1W}) &= {}_5C_4\,[P(\text{R})]^4\,[P(\text{W})]^1 \\ &= 5(0.25)^4\,(0.75)^1 \\ &\doteq 0.015 \text{ or } 1.5\% \end{aligned}$$

$$\begin{aligned} P(\text{5R and 0W}) &= {}_5C_5\,[P(\text{R})]^5\,[P(\text{W})]^0 \\ &= 1(0.25)^5\,(0.75)^0 \\ &\doteq 0.001 \text{ or } 0.1\% \end{aligned}$$

You can find the probabilities of the six outcomes in the answer to question c. of the previous example by expanding the binomial $[P(\mathrm{R})+P(\mathrm{W})]^5$.

$$[P(\mathrm{R})+P(\mathrm{W})]^5 = {}_5C_0[P(\mathrm{R})]^0[P(\mathrm{W})]^5 + {}_5C_1[P(\mathrm{R})]^1[P(\mathrm{W})]^4 + {}_5C_2[P(\mathrm{R})]^2[P(\mathrm{W})]^3 + {}_5C_3[P(\mathrm{R})]^3[P(\mathrm{W})]^2 + {}_5C_4[P(\mathrm{R})]^4[P(\mathrm{W})]^1 + {}_5C_5[P(\mathrm{R})]^5[P(\mathrm{W})]^0$$

Since $P(\mathrm{R})+P(\mathrm{W})=1$, $[P(\mathrm{R})+P(\mathrm{W})]^5=1$ and the sum of the six probabilities in the **probability distribution** is 1 or 100%. A probability distribution is a list of all the possible outcomes and the corresponding probabilities. The sum of the probabilities of a probability distribution is always 100%. This makes sense because if you were to answer the five-question test, it is certain you would get 0, 1, 2, 3, 4, or 5 answers correct.

In general, for binomial trials, you can find the terms of the probability distribution (or binomial distribution) by using a formula related to the general term of the Binomial Theorem.

If p is the probability of success and q is the probability of failure, then the probability of x successes in n trials is

$$P(x \text{ successes}) = {}_nC_x p^x q^{n-x}$$

Note: $q = 1 - p$

You will use this formula in the next example.

Mathematical Process

- Communication
- [x] Connection
- Estimation
- Mental Math
- Problem Solving
- Reasoning
- Technology
- Visualization

probability distribution: a distribution of probabilities across the outcomes of an experiment

Mathematical Process

- Communication
- Connection
- Estimation
- Mental Math
- Problem Solving
- [x] Reasoning
- Technology
- Visualization

Mathematical Process
- Communication
- Connection
- Estimation
- Mental Math
- Problem Solving
- Reasoning
- Technology
- Visualization

Example

Oksana plays baseball for one of the community league teams. Her batting average this season is 0.400.

a. What is the probability that Oksana will get only 1 hit in her next 3 times at bat?

b. Use your graphing calculator to determine the probability distribution for 0, 1, 2, and 3 hits. Graph this distribution on a histogram.

c. What is the probability that Oksana will get at least 1 hit in her next 3 times at bat?

Solution

a. Method 1: Using the Formula

Each time at bat, the probability of Oksana getting a hit is 0.400.

The probability of Oksana not getting a hit is

$$\begin{aligned} q &= 1 - p \\ &= 1 - 0.400 \\ &= 0.600 \end{aligned}$$

The number of binomial trials is the number of times Oksana is at bat. Therefore, $n = 3$.

The number of successes is the number of hits. Therefore, $x = 1$.

$$\therefore P(x \text{ successes}) = {}_nC_x p^x q^{n-x}$$

$$\begin{aligned} P(1 \text{ hit}) &= {}_3C_1 (0.400)^1 (0.600)^{3-1} \\ &= 3(0.400)^1 (0.600)^2 \\ &= 0.432 \text{ or } 43.2\% \end{aligned}$$

The probability that Oksana gets only one hit in her next three times at bat is 0.432 or 43.2%.

Method 2: Using a Graphing Calculator

Access the Binomial Probability Distribution feature.

2nd [DISTR] 0 (0:binompdf( )

To determine a particular term in the binomial distribution, you will need to enter the values of n, p, and x.

As in Method 1, the number of trials, n, is 3, the probability of success for each trial, p, is 0.400, and the number of successes in n trials, x, is 1.

3 , • 4 , 1) ENTER

The probability that Oksana gets only one hit in her next three times at bat is 0.432 or 43.2%.

Mathematical Process

- Communication
- Connection
- Estimation
- Mental Math
- Problem Solving
- Reasoning
- [x] Technology
- Visualization

Mathematical Process
- Communication
- Connection
- Estimation
- Mental Math
- Problem Solving
- Reasoning
- [X] Technology
- Visualization

b. Find the probability distribution for the number of hits when Oksana is at bat three times. The possible outcomes are 0, 1, 2, and 3 hits. Using the formula, the corresponding probabilities are ${}_3C_0\,(0.400)^0\,(0.600)^3$, ${}_3C_1\,(0.400)^1\,(0.600)^2$, ${}_3C_2\,(0.400)^2\,(0.600)^1$, and ${}_3C_3\,(0.400)^3\,(0.600)^0$.

As in the answer to question a., you can use the Binomial Probability Distribution feature on your graphing calculator. As before, $n = 3$ and $p = 0.400$; but the number of hits, x, is $\{0, 1, 2, 3\}$.

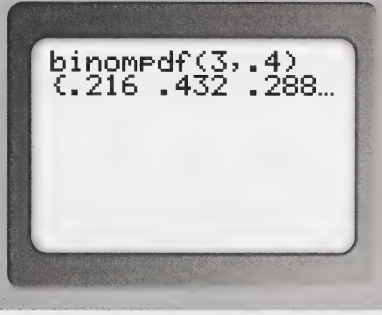
2nd [DISTR] 0 (0:binompdf() 3 , • 4 ,

2nd [{] 0 , 1 , 2 , 3 2nd [}])

ENTER

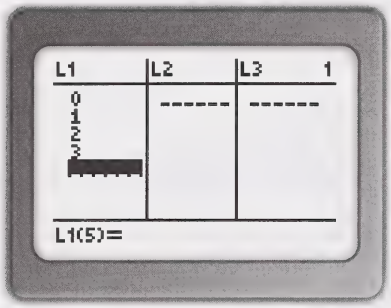
Note: You can abbreviate this method by simply pressing the following:

2nd [DISTR] 0 (0:binompdf() 3 , • 4) ENTER



The probability distribution corresponding to 0, 1, 2, and 3 hits respectively is {0.216, 0.432, 0.288, 0.064}.

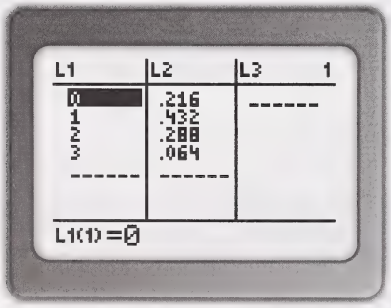
To graph the histogram illustrating this data, first enter 0, 1, 2, 3 in list L1.



Next, enter the data from the probability distribution as list L2.

Press [2nd] [QUIT] [2nd] [DISTR] [0] (0:binompdf() [3] [,] [•] [4] [)] [STO▸] [2nd] [L2] [ENTER]

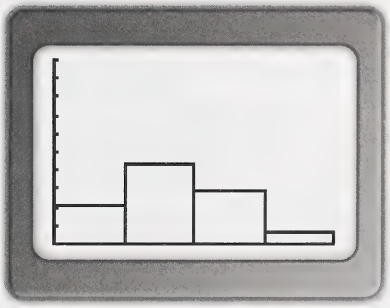
Press [STAT] [1] (1:Edit…) to view the lists.



Use the following window settings to display the histogram.

To display a histogram, press 2nd [STAT PLOT] 1. Make sure your display matches the following display.

Press GRAPH to display the histogram.

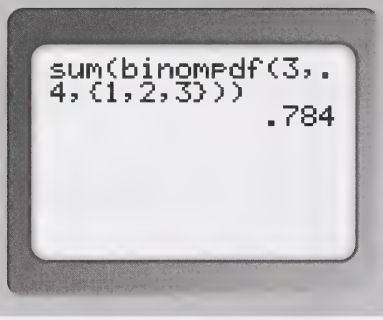


c. Method 1

$$\begin{aligned} P(\text{at least 1 hit}) &= P(1 \text{ hit}) + P(2 \text{ hits}) + P(3 \text{ hits}) \\ &= 0.432 + 0.288 + 0.064 \\ &= 0.784 \end{aligned}$$

You can also use your graphing calculator.

2nd [LIST] [Select the MATH menu.] 5 (5:sum() 2nd
[DISTR] 0 (0:binompdf() 3 , • 4 , 2nd
[{] 1 , 2 , 3 2nd [}])) ENTER



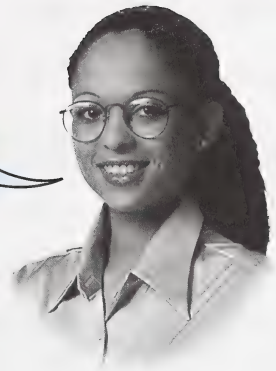
Method 2

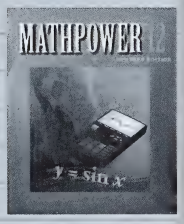
$$\begin{aligned} P(\text{at least 1 hit}) &= 1 - P(0 \text{ hits}) \\ &= 1 - {}_3C_0\,(0.400)^0\,(0.600)^3 \\ &= 1 - 0.216 \\ &= 0.784 \text{ or } 78.4\% \end{aligned}$$

The probability that Oksana will get at least one hit at her next three times at bat is 0.784 or 78.4%

Mathematical Process

- Communication
- Connection
- Estimation
- Mental Math
- Problem Solving
- Reasoning
- [X] Technology
- Visualization





Turn to page 405 of *MATHPOWER 12* and read from the top of the page to the red line on page 407, working through Examples 1 to 3.

2. Answer the following on pages 407 and 408 of the textbook.

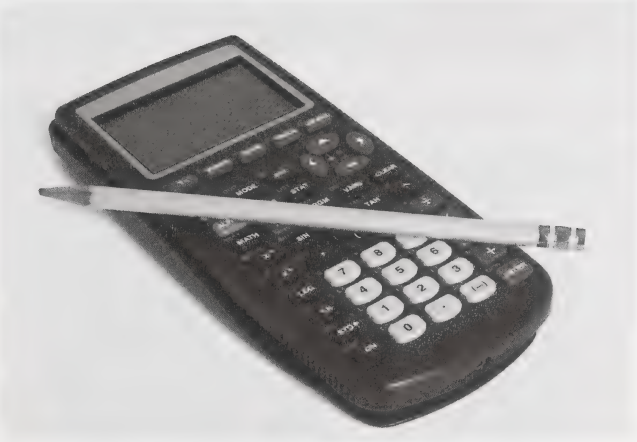
a. questions 3, 7, 8, 9, and 10 of "Practice"
b. questions 14, 15, 17, 19, 21, and 22 of "Applications and Problem Solving"

Turn to page 121 to compare your responses with those in the Appendix.

Mathematical Process

- Communication
- Connection
- Estimation
- Mental Math
- Problem Solving
- Reasoning
- [x] Technology
- Visualization

In Example 2 on page 406 of the textbook, you may have noticed that the graphing calculator solution uses the Binomial Cumulative Distribution feature. Study the following example to see how this feature can be used to your advantage when answering questions involving binomial distributions.



Example

A fair coin is tossed 5 times.

a. Determine the probability distribution for the number of heads.
b. What is the probability of obtaining no more than two heads?

Solution

a. $n = 5$ (number of tosses)

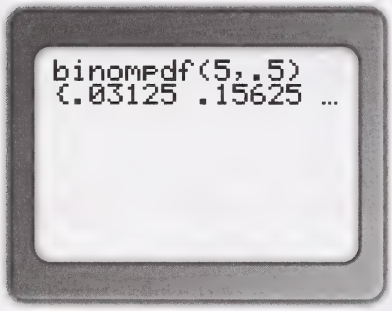
$x = 0, 1, 2, 3, 4, 5$ (possible number of heads)

$p = 0.5$ (chance of success, or tossing a head)

Use the Binomial Probability Distribution feature on your graphing calculator by pressing the following:

2nd [DISTR] 0 (0:binompdf() 5 , • 5)

ENTER



Note: Because you are finding the entire distribution, you do not have to enter the values of x.

The binomial distribution is summed up in the following table.

Number of Heads	Probability
0	0.031 25
1	0.156 25
2	0.3125
3	0.3125
4	0.156 25
5	0.031 25

Mathematical Process

- Communication
- Connection
- Estimation
- Mental Math
- Problem Solving
- Reasoning
- [x] Technology
- Visualization

b. Method 1: Using the Binomial Distribution

No more than 2 heads means getting 0, 1, or 2 heads.

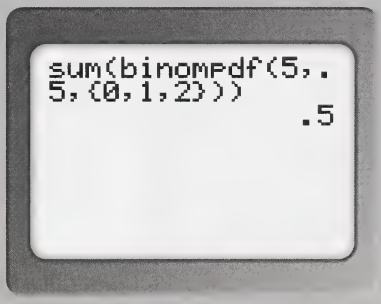
$$\begin{aligned}\therefore P(\text{no more than 2 heads}) &= P(0 \text{ heads}) + P(1 \text{ head}) + P(2 \text{ heads}) \\ &= 0.031\,25 + 0.156\,25 + 0.3125 \\ &= 0.500\,00\end{aligned}$$

The probability of obtaining no more than two heads is 0.5 or 50%.

You can also use the Sum feature and the Binomial Probability Distribution feature on your graphing calculator.

Since $n = 5$, $p = 0.5$, and $x = \{0, 1, 2\}$, press the following.

2nd [LIST] [Select the MATH menu.] 5 (5:sum() 2nd
[DISTR] 0 (0:binompdf() 5 , • 5 , 2nd
[{] 0 , 1 , 2 2nd [}])) ENTER



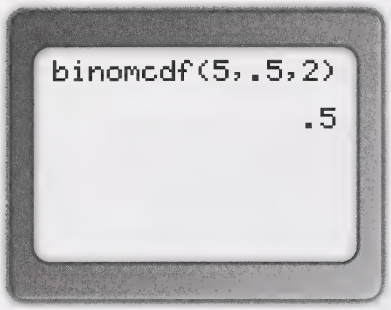
Method 2: Using the Binomial Cumulative Distribution Feature

This feature adds the probabilities in the binomial distribution up to and including the value of x.

$n = 5$
$p = 0.5$
$x = 2$ (no more than 2 heads)

Press the following:

2nd [DISTR] ALPHA [A] (A:binomcdf() 5 , • 5 , 2) ENTER



The probability of obtaining at least two heads is 0.5 or 50%.

Would you like to see an additional example explaining how to use your graphing calculator to determine binomial distributions? From the Pure Mathematics 30 Multimedia CD, work through the graphing calculator demonstration called *Binomial Probability Distribution.*

Now Try This

Histograms of binomial distributions are typically bell-shaped. Have you ever wondered how the shape of the histogram would change if the number of trials changes or the probability of a success, p, changes?

You can explore changes to the number of trials or changes to the probability using the applet titled *Binomial Coin Experiment* at the following website:

http://www.math.uah.edu/stat/bernoulli/index.xhtml

3. Using $p = 0.5$, how does the histogram change when the stop frequency (number of trials) changes from 10 to 100 and then to 1000?

4. Using a stop frequency of 100, how does the histogram change when p changes from 0.5 to 0.2?

Turn to page 135 to compare your responses with those in the Appendix.

Mathematical Process

- ☐ Communication
- ☒ Connection
- ☐ Estimation
- ☐ Mental Math
- ☐ Problem Solving
- ☐ Reasoning
- ☒ Technology
- ☐ Visualization

Mathematical Process
- [x] Communication
- [ ] Connection
- [ ] Estimation
- [ ] Mental Math
- [ ] Problem Solving
- [ ] Reasoning
- [ ] Technology
- [ ] Visualization

Looking Back

In this activity you explored binomial (or Bernoulli) experiments. Remember, for each trial of a binomial experiment, there are only two possible outcomes: success and failure. The probability of success, p, and the probability of failure, q, does not change throughout the experiment. You discovered that the probability of x successes in n trials is given by the formula $P(x \text{ successes}) = {}_nC_x\, p^x\, q^{n-x}$. You also discovered that you can change the values of x from 0 through n to obtain a distribution of probabilities that can be graphed using a histogram.

In your journal, create a multi-part exam question that covers the main ideas in this activity and prepare an answer key. Exchange questions with another student taking Pure Mathematics 30. Answer each other's question, and discuss your solutions.

Activity 2: The Mean and Standard Deviation

Mathematical Process
- [x] Communication
- [ ] Connection
- [ ] Estimation
- [ ] Mental Math
- [ ] Problem Solving
- [ ] Reasoning
- [ ] Technology
- [ ] Visualization

Fashions change over time. If you look at fashions that were current at the turn of the last century, they look plain and drab. However, to the people back then, these clothes were perfectly normal. Have a look in your own closet at home. Do you have any clothes you thought were the height of fashion when you bought them, but now wouldn't even think about wearing today?

Fashion experts claim that one of the main reasons styles change is that a person or a small group of influential people dare to dress in a way that deviates from the norm; then others copy them.

What is the norm or average? To what degree do items deviate or differ from the average? These are questions that are addressed not only by fashion critics, but also by statisticians.

In this activity you will explore how to determine the average or **mean** of a set of data. You will also investigate a measure, called **standard deviation**, that describes the extent to which data differs from the mean.

Turn to page 409 of *MATHPOWER 12* and read "The Mean and the Standard Deviation."

1. Answer the following on page 409 of the textbook.

a. "Explore: The Spread of Data"
b. questions 1 to 4 of "Inquire"

Turn to page 136 to compare your responses with those in the Appendix.

Given a set of data, $\{x_1, x_2, x_3, \ldots, x_n\}$, you can find the mean, $\bar{x}$, using the following formula:

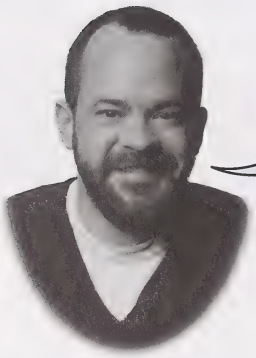
$$\bar{x} = \frac{x_1 + x_2 + x_3 + \ldots + x_n}{n} \quad \text{or} \quad \bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$$

The formula for the standard deviation, σ, of this set is as follows:

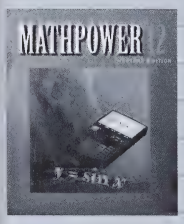
$$\sigma = \sqrt{\frac{(x_1 - \bar{x})^2 + (x_2 - \bar{x})^2 (x_3 - \bar{x})^2 + \ldots + (x_n - \bar{x})^2}{n}}$$

$$= \sqrt{\frac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{n}}$$

Note: The lower-case Greek letter sigma, σ corresponds to the lower-case letter "s," the first letter of the term standard deviation.



In Pure Mathematics 30, you will not need to memorize the formula for standard deviation of a set of data. You will be using your graphing calculator instead.



mean: the average obtained by dividing the sum of the data by the number of data in the set

standard deviation: the extent to which data differs from the mean

Together, what do the mean and standard deviation of a set of data tell you?

The standard deviation is a measure of how the data is clustered about the mean. For large sets of data, approximately 68.3% of the data lies within one standard deviation of the mean and approximately 95.4% of the data lies within two standard deviations of the mean.

Consider the following example.

Mathematical Process
- [x] Communication
- [x] Connection
- [ ] Estimation
- [ ] Mental Math
- [ ] Problem Solving
- [ ] Reasoning
- [ ] Technology
- [ ] Visualization

Example

On a province-wide mathematics exam, the mean was 65 and the standard deviation was 15. Describe the distribution of marks on the exam.

Solution

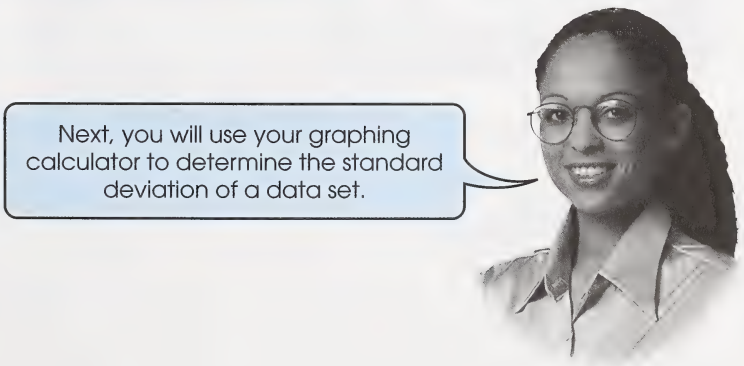
$\bar{x} = 65$ and $\sigma = 15$

$$\therefore \bar{x} - \sigma = 65 - 15 \quad \text{and} \quad \bar{x} + \sigma = 65 + 15$$
$$= 50 \qquad\qquad = 80$$

Approximately 68.3% of the students who wrote this exam had marks between 50 and 80.

$$\bar{x} - 2\sigma = 65 - 2(15) \quad \text{and} \quad \bar{x} + 2\sigma = 65 + 2(15)$$
$$= 35 \qquad\qquad = 95$$

Approximately 95.4% of the students who wrote this exam had marks between 35 and 95.

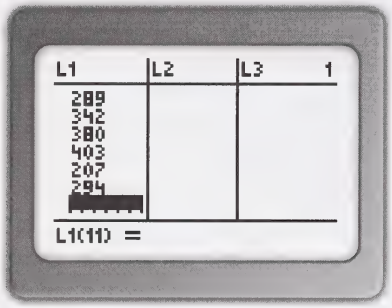


Example

At Alpine Village, the annual snowfall (in centimetres) for the last 10 years has been 321, 267, 232, 304, 289, 342, 380, 403, 207, and 294. What was the mean snowfall and the standard deviation? Round your answers to the nearest tenth.

Solution

Enter the data into your graphing calculator as a list. Press STAT 1 (1:Edit), and enter the data.



Next, press STAT [Select the CALC menu.] 1 (1:1–Var Stats) ENTER.

Note: The standard deviation is represented by σx.

The mean snowfall was 303.9 cm, and the standard deviation was approximately 58.0 cm.

Mathematical Process

- Communication
- Connection
- Estimation
- Mental Math
- Problem Solving
- Reasoning
- [x] Technology
- Visualization

If a number of data values are repeated, you can find the mean and standard deviation as in the preceding example or you can prepare two lists:

- The first list is for each distinct data value.
- The second list is for the frequency (number of times each data value occurs).

Example

Determine the mean and standard deviation of the data in the following table. Round your answers to the nearest tenth.

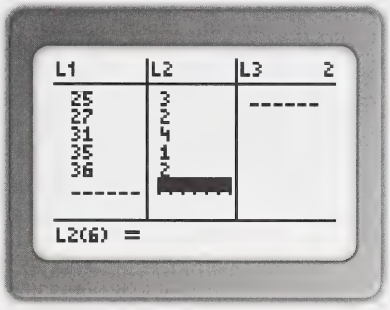
Data Value	Frequency
25	3
27	2
31	4
35	1
36	2

Mathematical Process

- ☐ Communication
- ☐ Connection
- ☐ Estimation
- ☐ Mental Math
- ☐ Problem Solving
- ☐ Reasoning
- ☒ Technology
- ☐ Visualization

Solution

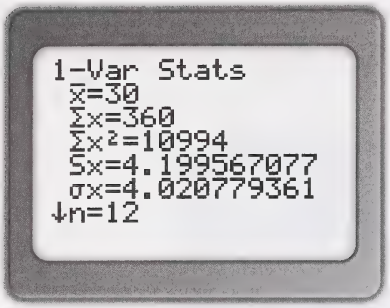
Enter the data values as list L1 and their corresponding frequencies as list L2.



Make sure each data value and its corresponding frequency appear in the same row.

After you have entered the lists, press the following.

STAT [Select the CALC menu.] 1 (1:1–Var Stats) 2nd [L1] , 2nd [L2]



Always check the value of n. It should be the sum of the frequencies. Here, $n = 12$.

The mean is 30, and the standard deviation is approximately 4.0.

Would you like to see an additional example explaining how to use your graphing calculator to determine a standard deviation? From the Pure Mathematics 30 Multimedia CD, work through the calculator demonstration called *Standard Deviation*.

Turn to pages 410 and 411 of *MATHPOWER 12* and work through Example 1.

2. Answer questions 1 and 3.a. of "Practice" on page 412 of the textbook.

Turn to page 137 to compare your responses with those in the Appendix.

Your mark on an exam is not always a good indication of how well you did relative to the rest of the class. To answer this question, you need to know the mean and standard deviation as well. If your mark is less than one standard deviation above or below the mean, you are in good company. (Remember, approximately 68.3% of the marks are within one standard deviation of the mean.)

The number of standard deviations a mark is above or below the mean is called its **z-score**. In the next set of questions you will use standard deviation to solve problems and be introduced to z-scores as a way of analyzing data.

z-score: the number of standard deviations a data value, x, lies from the mean

Turn to pages 411 and 412 of *MATHPOWER 12* and work through Examples 2 and 3.

3. Answer the following on page 413 of the textbook.

a. question 4 of "Practice"
b. questions 5, 7, and 11 of "Applications and Problem Solving"

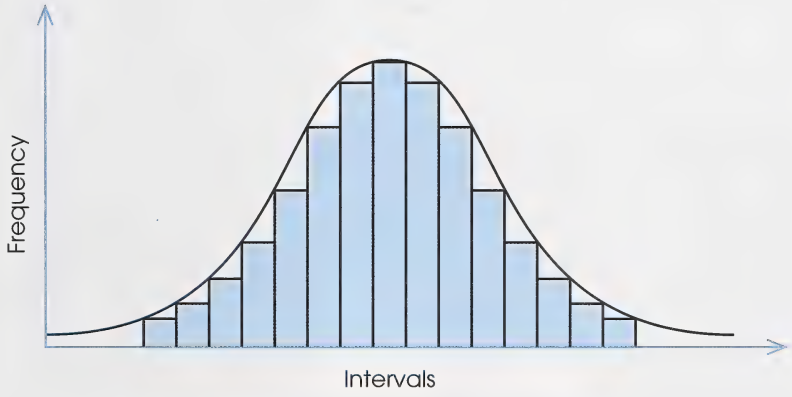
Turn to page 140 to compare your responses with those in the Appendix.

Mathematical Process

- ☐ Communication
- ☒ Connection
- ☐ Estimation
- ☐ Mental Math
- ☐ Problem Solving
- ☐ Reasoning
- ☐ Technology
- ☒ Visualization

Now Try This

A powerful way to visualize a set of data is to graph its frequency distribution as a histogram. Histograms that represent data (such as height, weight, mathematical achievement, and physical strength) are typically bell-shaped. If you look at the heights of students in grade 12, for example, most of the students will be clustered about the mean; however, there will be a few at the extremes.

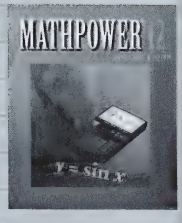


In the next activity you will explore bell curves in detail. To prepare for this exploration, answer the following questions.

Turn to "INVESTIGATING MATH" on page 414 of *MATHPOWER 12* and read "The Standard Normal Curve."

4. Answer question 1 of Investigation 1, "Relating a Normal Curve to the Standard Normal Curve."

Turn to page 143 to compare your response with the one in the Appendix.



Looking Back

In this activity you explored methods of finding the means and standard deviations of sets of data. You discovered that the standard deviation is a statistical measure of the spread or dispersion of the data around the mean.

Use your graphing calculator to obtain two sets of data with the same number of values in each set, the same mean, but with different standard deviations. Record these lists in your journal along with their means and standard deviations. Make a conjecture as to why these two sets have different standard deviations. Discuss your conjecture with another student taking Pure Mathematics 30. Together, create a third set of data that has the same number of values and the same mean as your first two sets, but with a larger standard deviation.

Mathematical Process

- [x] Communication
- [ ] Connection
- [ ] Estimation
- [ ] Mental Math
- [x] Problem Solving
- [ ] Reasoning
- [x] Technology
- [ ] Visualization

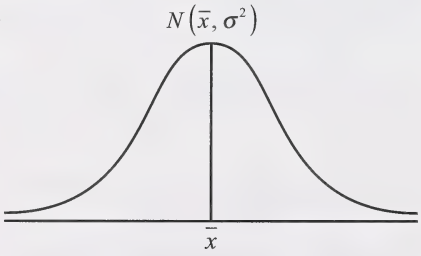
Activity 3: The Normal Distribution

Have you heard the term *bell curve* associated with physical and mental attributes, such as height, weight, strength, athletic ability, or intelligence? This term refers to the bell shape of the distribution of these attributes in a population. Take a person's skill level in chess, for example. Most participants are average players. The proportion of players declines as you deviate from the mean, either toward increasing skill on the right or toward decreasing skill on the left. Think of how many players are in the same category as those who have held the world championship—Boris Spassky, Robert Fischer, or Garry Kasparov.

WOLFGANG KAEHLER/CORBIS

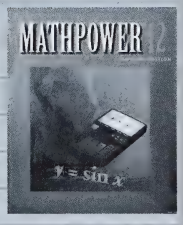
Bell-shaped distributions are approximated by the **normal distribution**, defined solely in terms of the mean, $\bar{x}$, and the standard deviation, σ. Normal distributions are denoted by $N(\bar{x}, \sigma^2)$. This function is defined as follows:

$$f(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{(x-\bar{x})^2}{2\sigma^2}}$$



normal distribution: a distribution of a set of data defined in terms of the mean and standard deviation

normal curve: the graph of a normal distribution



The graph of the preceding function is called the **normal curve**. You do not have to memorize this function, just how to use the distribution. Using the normal distribution is the focus of this activity.

The equation of the normal distribution was derived from the binomial distribution by Abraham de Moivre (1667–1754). If you have access to the Internet, you can find out more about Abraham de Moivre at the following website:

http://www-history.mcs.st-and.ac.uk/Biographies/De_Moivre.html

Turn to page 418 of *MATHPOWER 12* and read "The Normal Distribution."

1. Answer the following on page 418 of the textbook.

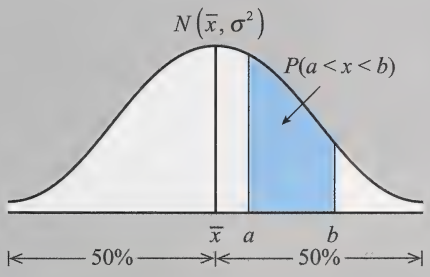
a. questions a. and b. of "Explore: Draw a Graph"
Note: The third row of the range should be 173–177, not 173–178.

b. questions 1 to 5 of "Inquire"

Turn to page 144 to compare your responses with those in the Appendix.

Now that you have been introduced to the normal curve, study its properties.

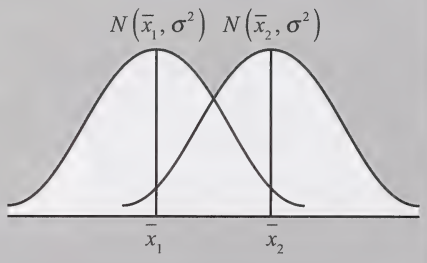
Properties of a Normal Distribution Curve

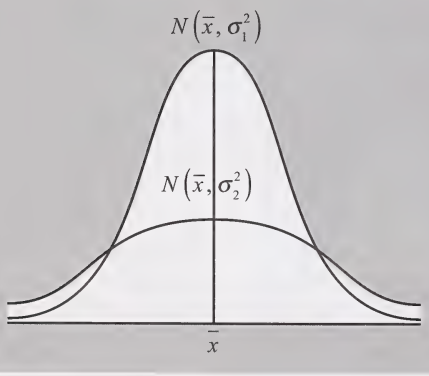


- The normal distribution is symmetric about the mean $\bar{x}$, ; 50% of the values lie above the mean (to the right), and 50% of the values lie below the mean (to the left). Because the mean divides the distribution in half, the mean is also the **median**. Because the maximum point on the graph occurs at the mean, the mean is also the **mode** of the distribution.

- Because 100% of the distribution lies below the curve, the total area below the curve is 100% or 1.
- Approximately 68.3% of the distribution lies within one standard deviation of the mean; that is, the area below the curve between $\bar{x} - \sigma$ and $\bar{x} + \sigma$ is about 0.683.
- Approximately 95.4% of the distribution lies within two standard deviations of the mean; that is, the area below the curve between $\bar{x} - 2\sigma$ and $\bar{x} + 2\sigma$ is about 0.954.
- The area below the curve between $x = a$ and $x = b$ is $P(a < x < b)$, the probability that a given x-value will lie between a and b.
- Distributions can have different means and different standard deviations.

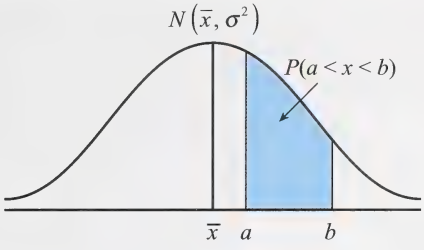
These distributions have the same standard deviation but different means: $\bar{x}_1 < \bar{x}_2$.





These distributions have the same mean but different standard deviations: $\sigma_1 < \sigma_2$.

How can you determine the area between $x = a$ and $x = b$?

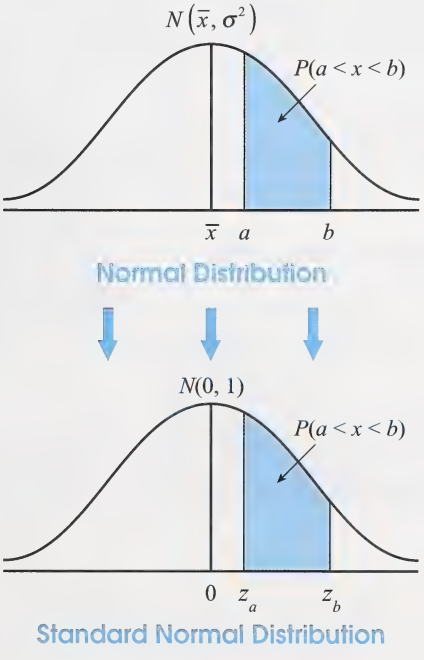


The simplest method—the one you are encouraged to use—is using the Normal Distribution feature of a graphing calculator. However, some models of graphing calculators do not have this feature. To circumvent this problem, the textbook and exams that test this concept contain tables for areas under the normal curve.

The data in tables have been standardized. It is not practical to have different tables corresponding to distributions with different means and different standard deviations. Instead, when using tables, you must first transform the normal distribution you are working with into the **standard normal distribution**, $N(0, 1)$, which is a normal distribution with a mean of 0 and a standard deviation of 1.

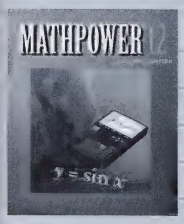
standard normal distribution: the normal distribution with a mean of 0 and a standard deviation of 1

This is accomplished by converting the x-axis of the normal distribution into z-scores.



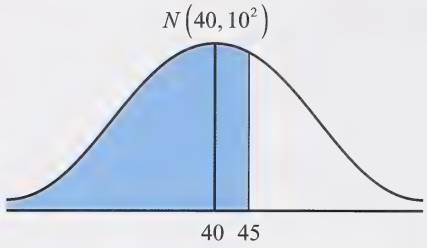
Example

Use the tables titled "Areas Under the Standard Normal Curve, to the left of z" on pages 416 and 417 of *MATHPOWER 12* to find the area (or percent of data) to the left of $x = 45$ and below the normal curve with a mean of 40 and a standard deviation of 10.



Solution

Sketch the distribution.



Use the formula $z = \frac{x-\bar{x}}{\sigma}$ to convert the mean, 40, and $x = 45$ into z-scores.

For the mean,

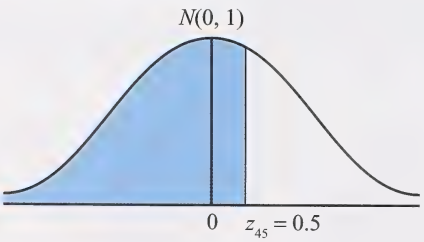
$$z = \frac{x-\bar{x}}{\sigma}$$
$$z_{40} = \frac{40-40}{10}$$
$$= 0$$

For $x = 45$,

$$z = \frac{x-\bar{x}}{\sigma}$$
$$z_{45} = \frac{45-40}{10}$$
$$= 0.5$$

A z-score of 0.5 means that a value of 45 lies 0.5 standard deviations above the mean, 40.

Sketch the standard normal distribution.

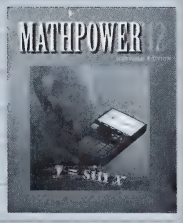


Because the z-score is positive, use the table on page 417.

	0.00	0.01	0.02
0.0	0.5000	0.5040	0.5080
0.1	0.5398	0.5438	0.5478
0.2	0.5793	0.5832	0.5871
0.3	0.6179	0.6217	0.6255
0.4	0.6554	0.6591	0.6628
0.5	0.6915	0.6950	0.6985

Therefore, the area below the curve and to the left of $x = 45$ is 0.6915. **Note:** You can express your answer as $P(x < 45) = 0.6916$ and interpret it as the probability of a given value in the distribution being less than 45.

Do the following questions to practise using tables to find areas under the standard normal curve.



Turn to "INVESTIGATING MATH" on pages 414 and 415 of *MATHPOWER 12* and read Investigation 2, "The Area Under the Standard Normal Curve."

2. Answer questions 1.a., 1.b., 1.c., 2.a., 2.b., 2.d., 2.e., 3.a., 3.b., 3.f., and 4 of Investigation 2, "The Area Under the Standard Normal Curve" on page 415 of the textbook.

Turn to page 146 to compare your responses with those in the Appendix.

Next, you will explore how to use a graphing calculator to determine areas under the normal curve.

Example

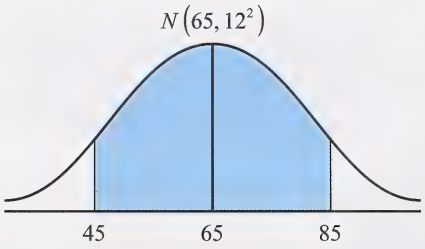
Use your graphing calculator to find the areas (percent of data) that satisfy the stated conditions for each normal distribution. Round each answer to 4 decimal places.

a. between 45 and 85, if the mean is 65 and the standard deviation is 12
b. greater than 67, if the mean is 60 and the standard deviation is 15
c. less than 70, if the mean is 63 and the standard deviation is 14

Mathematical Process
- Communication
- Connection
- Estimation
- Mental Math
- Problem Solving
- Reasoning
- [x] Technology
- Visualization

Solution

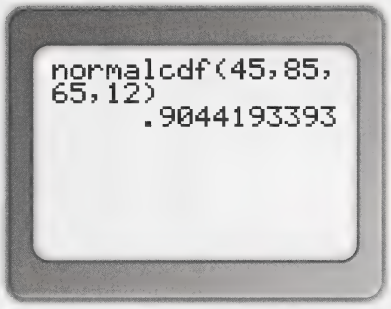
a. Sketch the distribution.



Use the Normal Distribution Probability feature (normalcdf) on your graphing calculator by pressing [2nd] [DISTR] [2] (2:normalcdf().

When you use this feature, you must enter the data as follows: normalcdf $(a, b, \bar{x}, \sigma)$. In this case, $a = 45$, $b = 85$, $\bar{x} = 65$, and $\sigma = 12$.

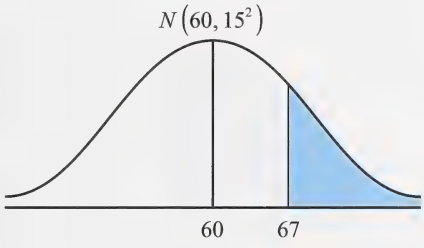
Therefore, press the following.



Note:
- a = *lower bound*
- b = *upper bound*
- $\bar{x}$ = *mean*
- σ = *standard deviation*

The area between 45 and 85 is approximately 0.9044.

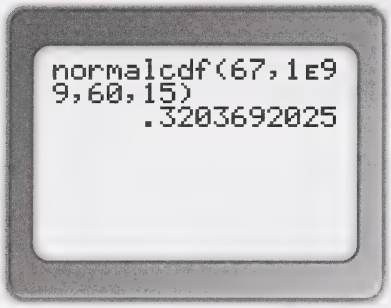
b. Sketch the distribution.



$\therefore a = 67$, $b = +\infty$, $\bar{x} = 60$, and $\sigma = 15$

Of course, you cannot enter $+\infty$ into your graphing calculator; however, 10^{99} is large enough for this calculation. You will have to enter it in exponential notation as "1E99."

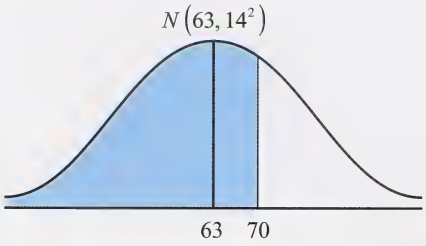
2nd [DISTR] 2 (2:normalcdf() 6 7 , 1 2nd [EE] 9 9 , 6 0 , 1 5) ENTER



$1E99 = 10^{99}$

The area greater than 67 is approximately 0.3204.

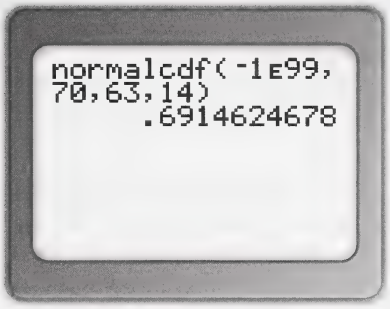
c. Sketch the distribution.



$\therefore a = -\infty$, $b = 70$, $\bar{x} = 63$, and $\sigma = 14$

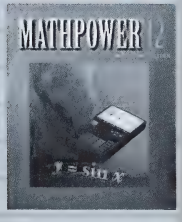
Of course, you cannot enter $-\infty$; however, -10^{99} is small enough for this calculation. You will have to enter it in exponential notation as "$-1\text{E}99$."

2nd [DISTR] 2 (2:normalcdf() (–) 1 2nd [EE] 9
9 , 7 0 , 6 3 , 1 4) ENTER



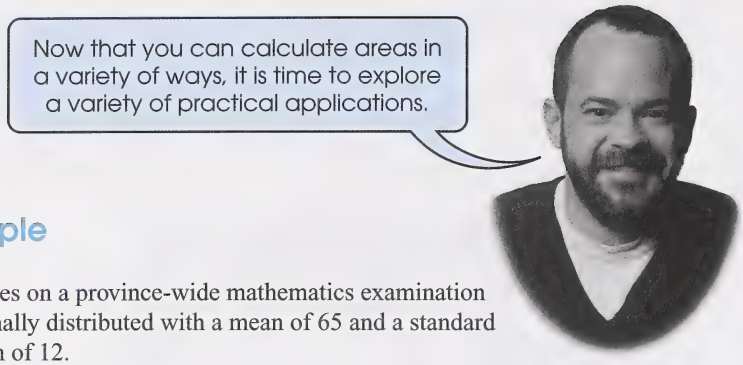
The area less than 70 is approximately 0.6915.

From the Pure Mathematics 30 Companion CD, open *The Normal Distribution Explorer* from the Explorers folder. This explorer allows you to calculate areas under the normal curve simply by entering the lower and upper bounds. Confirm the values in the preceding example using this explorer.



3. Turn to page 422 of *MATHPOWER 12* and answer questions 1, 2, 4, and 6 of "Practice."

Turn to page 148 to compare your responses with those in the Appendix.



Mathematical Process

- Communication
- Connection
- Estimation
- Mental Math
- [x] Problem Solving
- Reasoning
- [x] Technology
- Visualization

Example

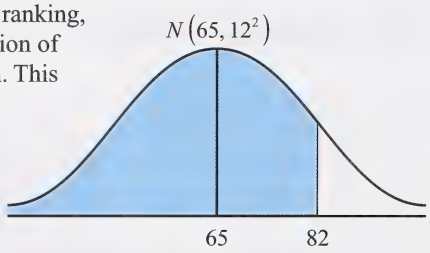
The scores on a province-wide mathematics examination are normally distributed with a mean of 65 and a standard deviation of 12.

a. Mary's mark on the exam is 82. What is her percentile ranking?
b. Muhammed's mark is at the 80th percentile. What is his mark on the exam?

Solution

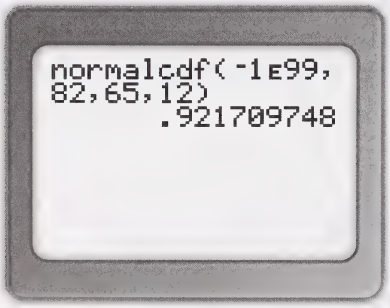
a. Sketch the distribution.

To determine Mary's percentile ranking, you must determine the proportion of students Mary beat on the exam. This proportion is the area below the normal curve and less than 82.



Use your graphing calculator to determine this area.

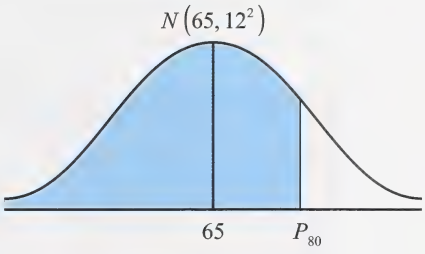
$a = -1\text{E}99$, $b = 82$, $\bar{x} = 65$, and $\sigma = 12$.



Since a mark of 82 is better than approximately 92% of the distribution, a mark of 82 is at the 92nd percentile. **Note:** This may be written as $82 = P_{92}$.

b. Sketch the distribution.

In this example, you must find the mark that corresponds to the 80th percentile, P_{80}. This mark is better than 80% of the distribution. Therefore, the area to the left of the required value is 0.80.



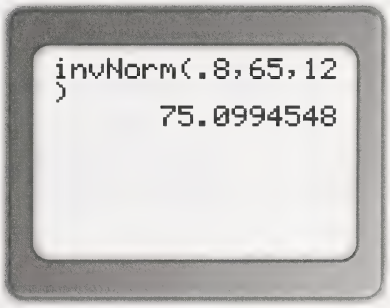
You must work backward from the area.

Method 1: Using a Graphing Calculator

You will need to use the Inverse Normal Distribution feature (invNorm) on your graphing calculator. Press [2nd] [DISTR] [3] (3:invNorm()

This feature requires the area lying to the left, the mean, and the standard deviation. The data should be entered as follows: invNorm(area, $\bar{x}$, σ). In this case, area = 0.8, $\bar{x} = 65$, and $\sigma = 12$.

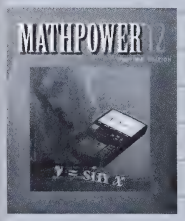
[•] [8] [,] [6] [5]
[,] [1] [2] [)] [ENTER]



Muhammed's mark on the exam is 75.

Method 2: Using Tables

Using the tables on pages 416 and 417 of *MATHPOWER 12*, look for an area closest to 0.8000.



	0.00	0.01	0.02	0.03	0.04	0.05
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023

The corresponding z-score is approximately 0.84.

$$\therefore\ z = \frac{x - \bar{x}}{\sigma}$$

$$0.84 \doteq \frac{P_{80} - 65}{12}$$

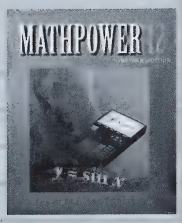
$$12(0.84) \doteq P_{80} - 65$$

$$P_{80} \doteq 12(0.84) + 65$$

$$\doteq 75.08$$

Muhammed's mark on the exam is 75.

Would you like to see additional examples explaining how to use your graphing calculator to solve problems involving normal distributions? From the Pure Mathematics 30 Multimedia CD, work through the calculator demonstrations called *Normal Cumulative Distribution* and *Inverse Normal Distribution*.



Turn to pages 419 to 422 of *MATHPOWER 12* and work through Examples 1 and 2.

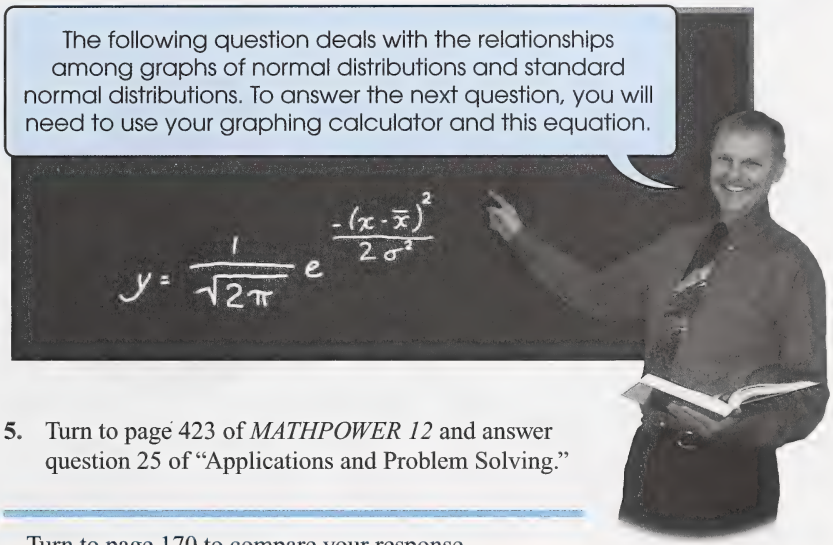
4. Answer the following on pages 422 and 423 of the textbook.

a. questions 7, 9, 10, 13, 14, and 16 of "Practice"
b. questions 18, 20, 21, 22, and 24.a. of "Applications and Problem Solving"

Turn to page 153 to compare your responses with those in the Appendix.

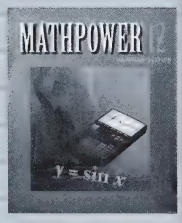
Mathematical Process
- Communication
- Connection
- Estimation
- Mental Math
- [x] Problem Solving
- Reasoning
- Technology
- [x] Visualization

Now Try This


The following question deals with the relationships among graphs of normal distributions and standard normal distributions. To answer the next question, you will need to use your graphing calculator and this equation.
$y = \frac{1}{\sqrt{2\pi}} e^{\frac{-(x-\bar{x})^2}{2\sigma^2}}$

5. Turn to page 423 of *MATHPOWER 12* and answer question 25 of "Applications and Problem Solving."

Turn to page 170 to compare your response with the one in the Appendix.



Mathematical Process
- [x] Communication
- Connection
- Estimation
- Mental Math
- Problem Solving
- Reasoning
- Technology
- Visualization

Looking Back

In this activity you explored the normal distribution and the standard normal distribution. You studied their graphs and their application to everyday problem situations using technology and tables.

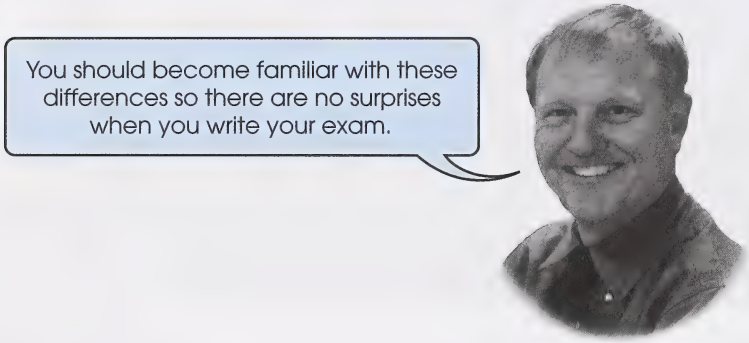
You made extensive use of the power of the graphing calculator. Occasionally, people criticize the use of graphing calculators in mathematics. In your journal, discuss the merits of using a graphing calculator. Does the graphing calculator enhance or hinder your understanding?

Follow-up Activities

If you had difficulties understanding the concepts and skills in the activities, it is recommended that you do the Extra Help. If you have a clear understanding of the concepts and skills, it is recommended that you do the Enrichment. You may decide to do both.

Extra Help

Come exam time, some students have difficulty when formulas they expect to see are presented in alternate forms or when symbols different from those with which they are familiar appear in those formulas. Even though mathematics is a highly standardized symbolic language, there are differences among users. These differences are often simply a matter of personal preference.



In this module, for example, you used $\bar{x}$ to represent the mean of a standard distribution. Many textbooks use μ instead.

Example

Write the z-score formula using μ instead of $\bar{x}$.

Solution

$$z_x = \frac{x - \bar{x}}{\sigma}$$

$$z_x = \frac{x - \mu}{\sigma} \quad \leftarrow \text{Substitute } \mu \text{ for } \bar{x}.$$

1. Write the more familiar form of each of the following formulas.

a. $\sigma = \sqrt{np(1-p)}$ **b.** $\mu = np$ **c.** $\sigma = \sqrt{\dfrac{\sum_{i=1}^{n}(x_i - \mu)^2}{n}}$

2. Turn to page 442 of *MATHPOWER 12* and answer questions 5, 10, and 11.a. of "Chapter Check." **Note:** You may wish to practise using the alternate forms from question 1.

Turn to page 172 to compare your responses with those in the Appendix.

Enrichment

In this module you studied both binomial distributions and normal distributions. Statisticians use many other distributions, because data cannot always be modelled by a normal distribution. Part of a statitician's job is to determine which distribution best represents the data. Knowledge of the subject matter in which they are gathering data is important. If you look on your calculator under the DISTR menu, you will see the names of some of the other distributions.

Another distribution that is quite similar to the binomial distribution is the **geometric distribution**.

Turn to page 441 of *MATHPOWER 12* and read "Exploring Math: The Waiting Period."

Answer questions 1, 2, and 3.a. of "Exploring Math: The Waiting Period" on page 441 of the textbook.

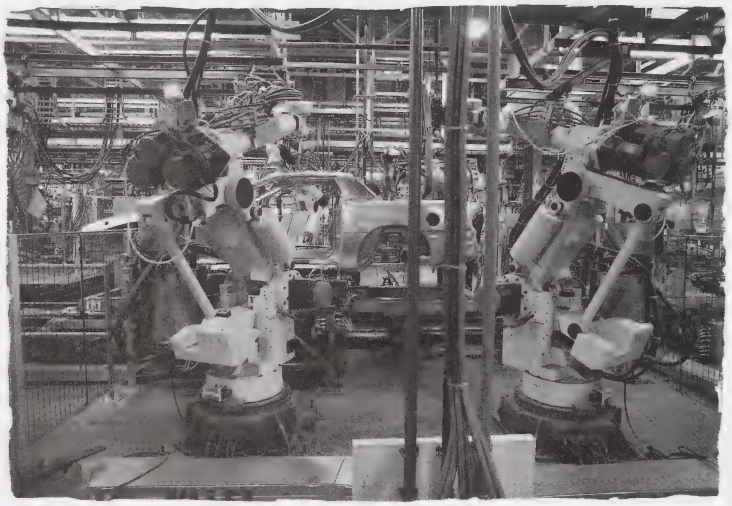
Turn to page 179 to compare your responses with those in the Appendix.

If you continue your study of statistics, you will study other distributions and the tests to determine which distribution best represents the data.

Conclusion

In this section you first explored probability problems using the binomial distribution as applied to small samples. You then explored the normal distribution as a generalization of the binomial distribution. You discovered that the normal distribution typically applies to large data sets, such as heights, weights, and physical and mental attributes of a large number of people. You used technology to determine means and standard deviations for these distributions, and you solved a variety of practical statistical problems modelled by normal distributions. For example, you calculated z-scores—using area tables for the standard normal distribution or a graphing calculator—to determine percentile rankings for students on exams.

The statistical analysis of data is important to a wide spectrum of everyday, professional, and business activities. In the automobile industry, the knowledge of normal distributions for the durability of the components that go into their vehicles is critical to providing quality products and to providing the warranties that protect the consumer.



Assignment

Turn to Assignment Booklet 7B and complete the assignment for Section 2.

Module Summary

In Section 1 you explored probability. You began by finding probabilities of simple events taken from small sample spaces. You built on this background to analyze compound events for large sample spaces. As part of your probability toolkit, you used Venn diagrams and tree diagrams, the multiplication and addition rules, and combinatorics theory.

In Section 2 you built on your work in probability to explore binomial experiments. You generalized these investigations to large samples, employing normal distributions to problem solve. You discovered that tables for standard normal distributions or built-in statistical features on your graphing calculator were necessary to solve these problems.

Knowledge of probability and statistics is an important skill in a modern technological society. You are constantly being bombarded at home and in the workplace by statistics presented by governments, business, and the media. Informed decision making can only occur if you can weigh the merits of the information and accept what is valid and reject information based on shoddy methods of collection and analysis.

Final Module Assignment

Turn to Assignment Booklet 7B and complete the final module assignment. Remember to submit your completed Assignment Booklet for assessment.

Diploma Examination Information

Preparing for the Diploma Examination

Congratulations! You have worked hard to get to this point in the course. You have covered a variety of topics and deserve the opportunity to demonstrate the skills you developed and the material you have mastered. Are you ready for the diploma examination?

To help you prepare for the diploma examination, go to the Alberta Education website:

http://www.education.gov.ab.ca

The directory "Kindergarten to Grade 12" posts course expectations, the makeup of the diploma examination, keyed copies of past examinations, preparation guides, and calculator policies. If you have access to the Internet, download copies of these documents or ask your teacher to help you obtain them. The following is a summary of some of the preparation strategies contained in these documents and suggested by the same people who put together the diploma examination you will be writing.

Some of these strategies are also contained in the video *Math Factor Review for Senior High Math, Volume 3*. This video may be purchased from the Learning Resources Centre. The video includes interviews with students, teachers, and examination personnel.

Here are a few strategies to help prepare for the examination:

- One of the most effective ways of preparing for the diploma examination starts right at the beginning of the course. You should develop your problem-solving and communication skills through daily practice and regular review.

- Find out what will be expected of you on the exam. Work through copies of previous exams. Pay attention to the format and unit weightings. At the time this course was written, the diploma examination consisted of 33 multiple-choice items, 6 machine-scored, numerical-response questions, and 3 written-response questions. The multiple-choice and numerical-response questions were worth 65% of the examination; two of the written-response questions were worth 10% each, and the third written-response question was worth 15% of the examination. Check the Alberta Education website or ask your teacher if this blueprint has changed.

- One of the written-response questions, worth 10%, is based on the student project. Make certain you are familiar with the solution, processes, and procedures contained in the project. Be prepared to address these aspects when you answer this question.

- The questions that appear on the diploma examination have been designed to test the extent to which you know the content and can demonstrate problem-solving skills. Access the website or ask your teacher how you can obtain copies of the curriculum standards. These standards not only outline the content that will be tested, but also describe the depth to which it will be tested as well. Find out what you will need to know and do to achieve an acceptable standard (50% or higher) and a standard of excellence (80% or higher).

- At least two weeks before the examination, prepare a review schedule. Divide the course into topics and allot time for an adequate review of each topic. It makes sense to spend more time on topics that you may have forgotten, covered earlier in the year, or found difficult. Stick to your schedule as much as possible. There may be times when you cannot adhere to it. Build in some flexibility into the schedule to allow for emergencies, and plan to complete your review at least one full day before the test so you can relax and brush up on a few minor points.

- Check and double check the time and place you will be writing the examination. Also, become familiar with the examination rules and policies. Make sure you have an approved graphing calculator for the examination.

- Find examples of each type of question. Practise these question types. Also, become familiar with the "directing words," such as *compare*, *describe*, *evaluate*, *explain*, *interpret*, *justify*, *prove*, and *solve*. These words are defined for you in information bulletins and tell you how you are expected to answer diploma examination questions that contain these words.

- Make summaries and outlines. Highlight key words and concepts. Develop memory aids to help you recall facts and procedures.

Writing the Pure Mathematics 30 Diploma Examination

Here are a few strategies that will help you do your best on the examination:

- Get a good night's sleep before the examination. You will do better if you are well rested and relaxed.
- Pack everything you will need the night before. Do not forget your graphing calculator! Remember, all memory locations must be cleared before you enter the examination room. Check your manual or ask your teacher how to clear your calculator's memory. Also, bringing a spare scientific calculator is not a bad idea. You will need soft lead pencils, an eraser, and a clear plastic ruler. Be prepared; bring tissues, cough drops, or candies, as required.
- Arrive a few minutes early so you are not rushed and so you enter the exam room in time for all instructions.
- At the beginning of the examination, look over the entire test. Answer questions you are comfortable with first. These questions may jog your memory and help you answer the questions you felt were more difficult.
- Keep track of the time and pace yourself. Remember, you have 3 h to complete the exam; but you should aim for $2\frac{1}{2}$ h so you can use the last $\frac{1}{2}$ h to make any necessary revisions.
- Answer every question. There is no penalty for guessing, and no answer is ever considered foolish. Instead, it is foolish to leave a question unanswered, especially if part marks are awarded!
- If you are stuck on a multiple-choice item, identify the choices you know are incorrect and choose from the remaining choices using logic and determining what is reasonable.
- In a difficult question, you will find it helpful to highlight key words to help clarify the question and to jog your memory.

- Label diagrams on the examination with the letters and numbers given in the question. Remember, most diagrams are drawn to scale; so, you can use your ruler to identify reasonable choices.

- Don't look for patterns in the multiple-choice section. Have a good reason for changing an answer; often, your first inclination is correct.

- When answering a numerical-response question, remember that answers are never negative. If you get a negative answer, try the question again.

- If you have time, prepare an outline for a written-response question and use the outline as a guide in answering the question.

- When completing a written-response question, keep in mind the reader of your response. The marker will want to know how well you understand the problem, can use the problem-solving strategies and mathematics involved, and can communicate your ideas.

- Often, rewriting the statement in a written-response question is a good way to begin. Don't forget to write a summary statement at the end of your solution and to check that you have addressed all aspects of the question.

Where Can I Obtain More Information About Pure Mathematics 30?

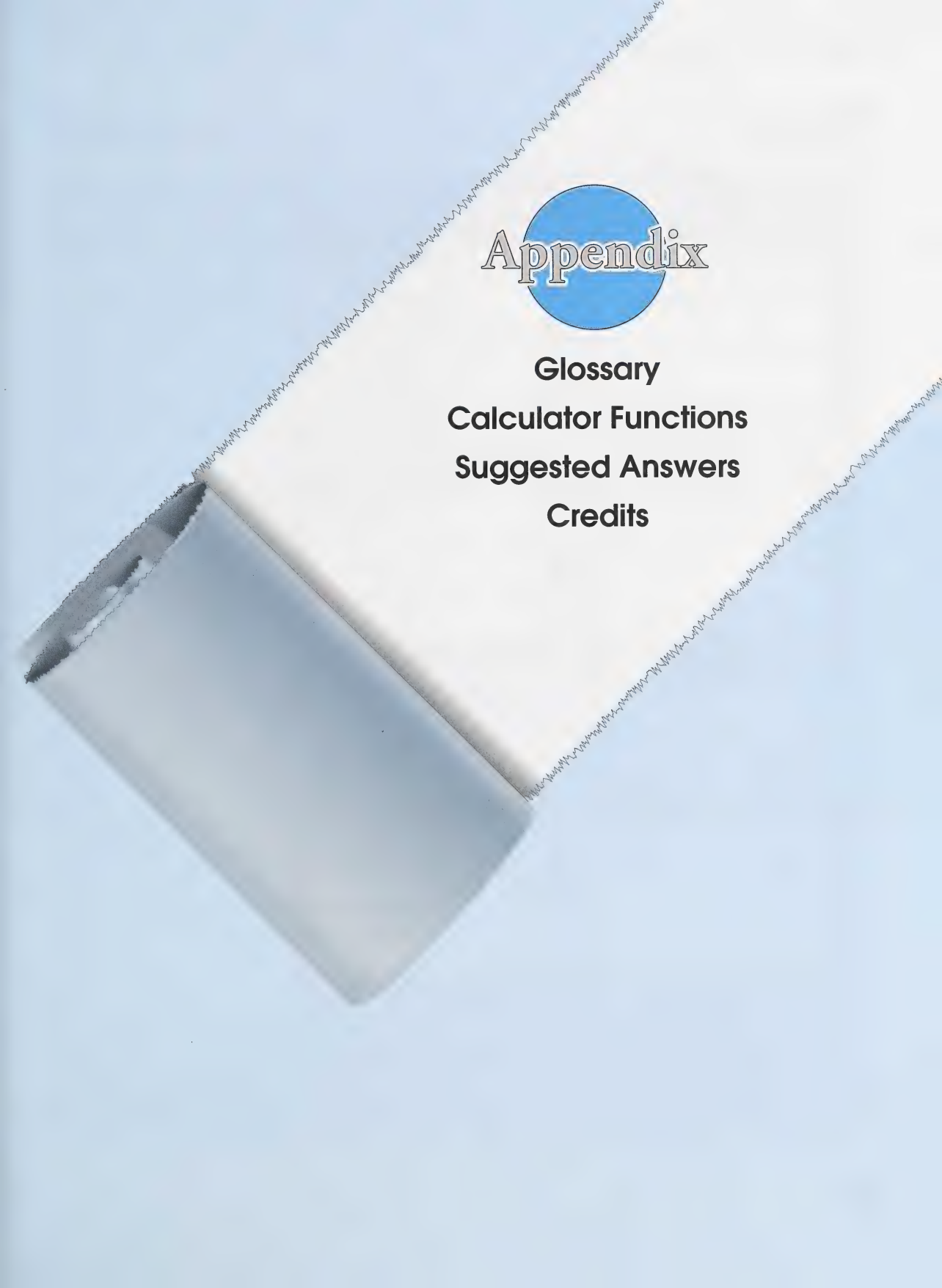
Alberta students will write a diploma examination at the end of the course. Alberta Education provides several documents to help students prepare for this examination. These documents are found in the directory under "Kindergarten to Grade 12" on the Alberta Education website, **http://www.education.gov.ab.ca**. Information like course expectations, the makeup of the diploma examination, keyed copies of previous examinations, preparation guides, and calculator policies are available to students at this site.

Each year, in February and September, Alberta Education provides teachers with information on a **student project**, which teachers **may** use as part of your overall assessment. Information to students will also be posted on the Alberta Education website. Check with your teacher to determine what you will be expected to do. Be aware that one of the diploma examination's written-response questions is worth 10% of your diploma examination mark and will deal with elements of this project.

You should take advantage of the many sources of information about Pure Mathematics 30. Your success depends on your understanding of course expectations and evaluation procedures. Work closely with your teacher and do not hesitate to ask questions.

Remember, take the initiative to find out all you can about Pure Mathematics 30.

Notes



Glossary

Calculator Functions

Suggested Answers

Credits

Glossary

bell curve: the normal curve

Bernoulli experiment: an experiment consisting of Bernoulli trials

Bernoulli trial: an event that has only two possible outcomes: success and failure

binomial distribution: a distribution of probabilities for Bernoulli trials

binomial experiment: a Bernoulli experiment

binomial trial: a Bernoulli trial

certain event: an event that always occurs

complementary event: written $\overline{A}$, an event consisting of all the outcomes in the sample space not in event A

dependent event: an event that is affected by the occurrence of other events

event: a set of outcomes of an experiment; a subset of a sample space

experiment: an action that has measurable or quantifiable results

favourable outcomes: the elements that comprise an event

impossible event: an event that never occurs

independent event: an event that is not affected by the occurrence of other events

mean: the average obtained by dividing the sum of the data by the number of data in the set

mutually exclusive events: events that cannot occur at the same time; events with no outcomes in common

normal curve: the graph of a normal distribution

normal distribution: a distribution of a set of data defined in terms of the mean and standard deviation

outcome: the result of a trial of an experiment; an element of a sample space

probability: the study of chance and uncertainty

probability distribution: a distribution of probabilities across the outcomes of an experiment

probability of an event: the number of favourable outcomes divided by the total number of outcomes in the sample space

probability tree: a tree diagram that includes probabilities

sample space: the entire set of outcomes possible

standard deviation: the extent to which data differs from the mean

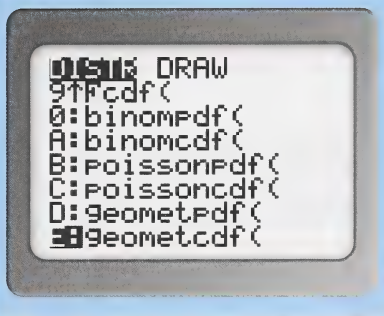
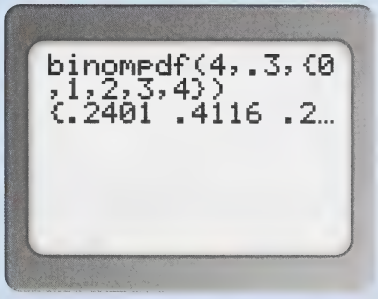
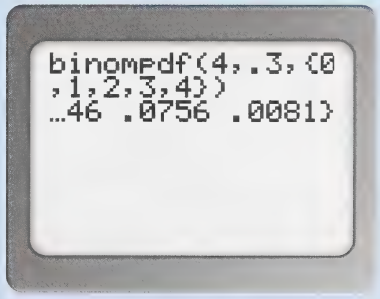
standard normal distribution: the normal distribution with a mean of 0 and a standard deviation of 1

Venn diagram: a diagram that uses overlapping circles in a rectangle to illustrate relationships among sets

z-score: the number of standard deviations a data value, x, lies from the mean

$$z_x = \frac{x - \bar{x}}{\sigma}$$

Calculator Functions

Calculator Function	Displays
Binomial Probability Distribution Feature 2nd [DISTR] 0 (0:binompdf()	
Determining a Probability Distribution e.g., the probability distribution for the number of hits a player batting 0.300 gets in the next 4 times at bat **1:** Access the Binomial Distribution feature. 2nd [DISTR] 0 (0:binompdf() **2:** Enter the number of trials, n, the probability of success for each trial, p, and the number of successes in n trials, x. 4 , • 3 , 2nd [{] 0 , 1 , 2 , 3 , 4 2nd [}]) **3:** Press ENTER to determine the probability distribution.	 $n = 4$ $p = 0.300$ $x = 0, 1, 2, 3, 4$  

Graphing a Probability Distribution

e.g., using example in Determining a Probability Distribution

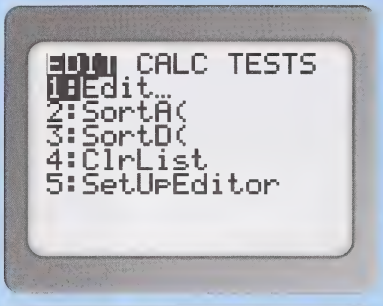
1: Press [STAT] [1] (1:Edit…), and enter 0, 1, 2, 3, 4 in list L1.

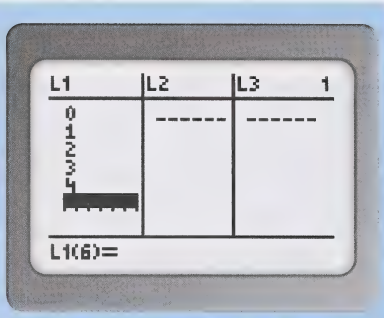
2: Enter the data from the probability distribution in list L2.

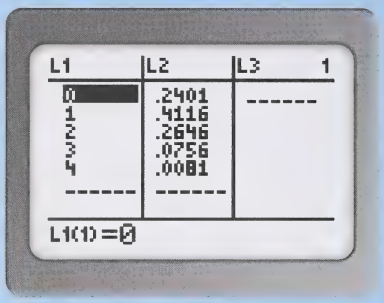
[2nd] [QUIT] [2nd] [DISTR] [0] (0:binompdf() [4] , [•] [3] , [2nd] [{] [0] , [1] , [2] , [3] , [4] [2nd] [}] [)] [STO▸] [2nd] [L2] [ENTER]

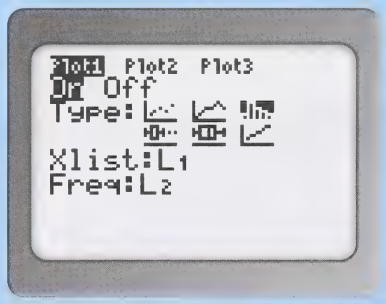
3: Press [2nd] [STAT PLOT] and adjust the settings to display a histogram using the information in lists L1 and L2.

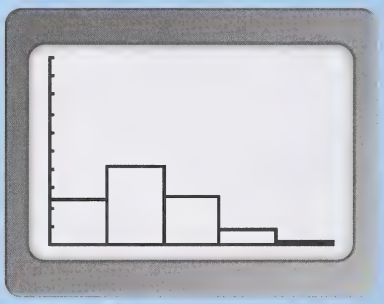
4: Press [GRAPH] to display the histogram.









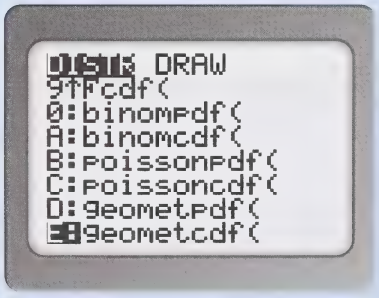


Window Settings:

Xmin = 0	Ymin = 0
Xmax = 5	Ymax = 1
Xscl = 1	Yscl = 0.1

Binomial Cumulative Distribution Feature

[2nd] [DISTR] [ALPHA] [A] (A:binomcdf()



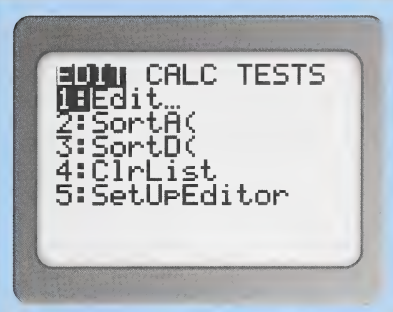
Calculating Mean and Standard Deviation (Individual Data)

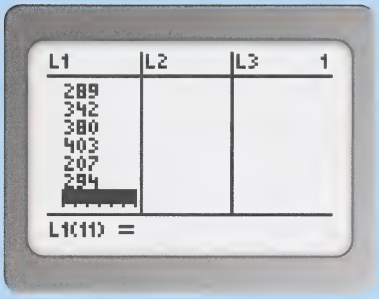
1: Press STAT 1 (1:Edit…), and enter the data into list L1.

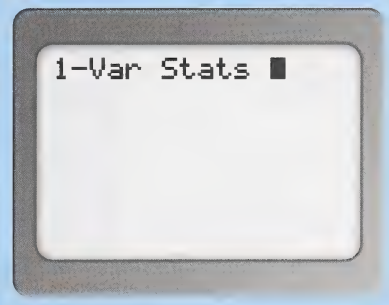
2: Once the data is entered, press

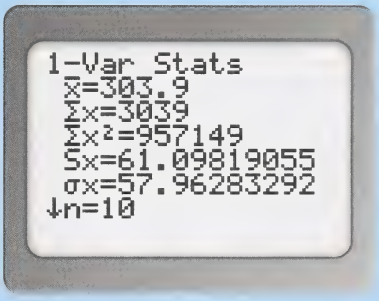
STAT [Select the CALC menu.] 1 (1:1-Var Stats)

3: Press ENTER to display the information.









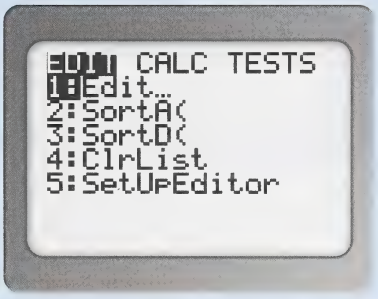
Calculating Mean and Standard Deviation (Grouped Data)

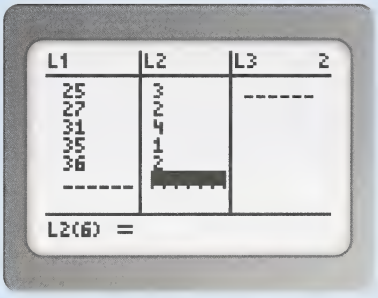
1: Press STAT 1 (1:Edit…), and enter the data into lists L1 and L2.

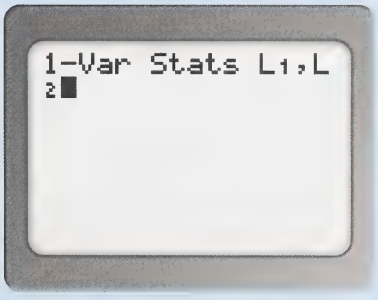
2: Once the data is entered, press the following:

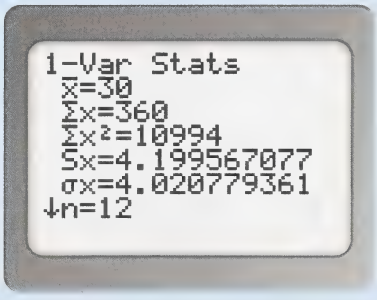
STAT [Select the CALC menu.] 1 (1:1-Var Stats) 2nd [L1] 2nd , 2nd [L2]

3: Press ENTER to display the information.









Finding Percent of Data of a Normal Distribution

e.g., between 45 and 85, if mean is 65 and standard deviation is 12

1: Select the Normal Distribution Probability feature.

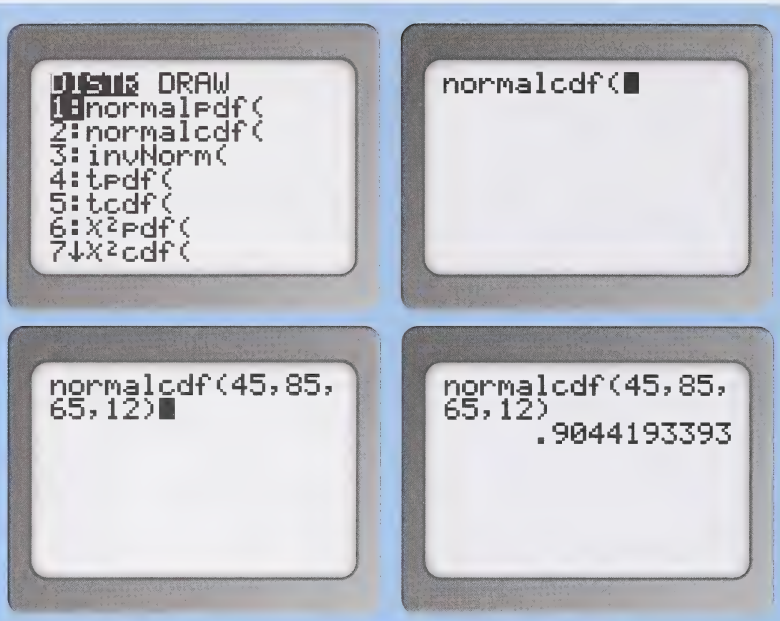
2nd [DISTR] 2

(2:normalcdf()

2: Enter the lower bound, upper bound, mean, and standard deviation in that order.

4 5 , 8 5 ,
6 5 , 1 2)

3: Press ENTER to find the area.



If lower bound is $-\infty$ or the upper bound is $+\infty$, enter the exponential notation "1E99" (10^{99}) in place of ∞ by pressing the following:

1 2nd [EE] 9 9

Finding a Certain Value of a Normal Distribution

e.g., a mark at the 75th percentile where the mean mark is 60 and the standard deviation is 10

1: Select the Inverse Normal Distribution feature.

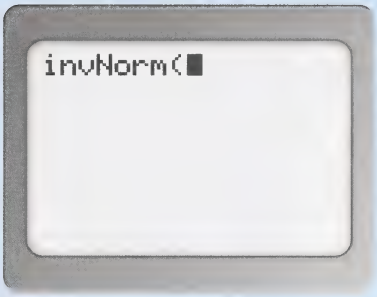
2nd [DISTR] 3

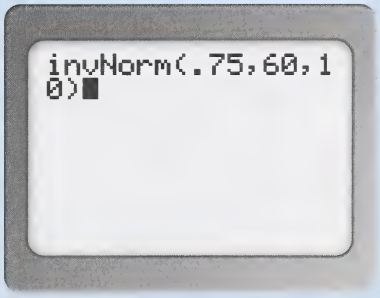
(3:invNorm()

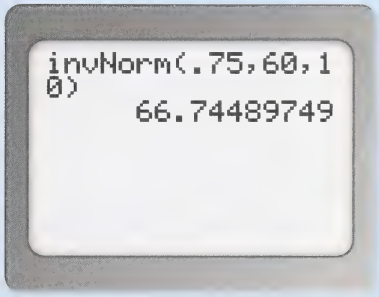
2: Enter the area lying to the left, mean, and standard deviation in that order.

7 5 , 6 0 ,
1 0)

3: Press ENTER to find the desired value.







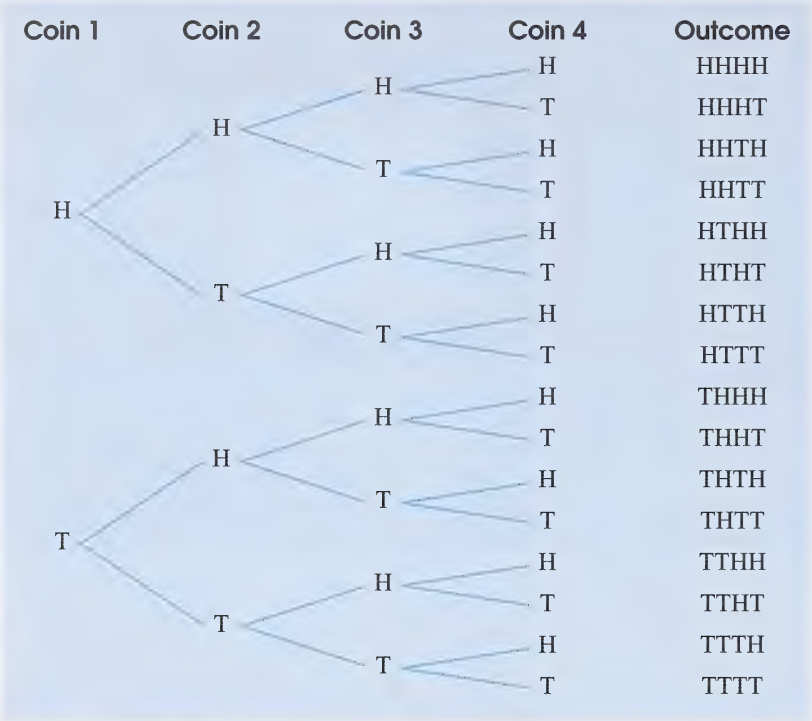
Note: The Inverse Normal Distribution feature requires you to enter the area lying to the left of the value. If you are given the area lying to the right, subtract it from 100 to get the area lying to the left and enter that value into the calculator.

Suggested Answers

Section 1: Activity 1

1. a. Textbook question 1 of "Explore: Use a Diagram," p. 368

1. The tree diagram shows all the possible outcomes of tossing four coins.



There are 16 possible outcomes.

b. Textbook question 1 of "Inquire," p. 368

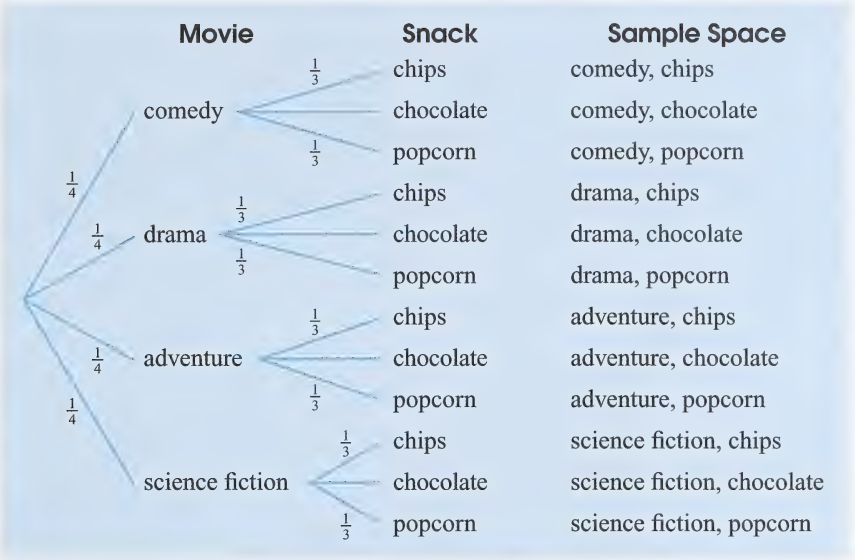
1. **a.** The total number of possible outcomes is $2 \times 2 \times 2 \times 2 = 16$, since each coin can turn up two ways.

   b. There are 5 outcomes with at least three heads: HHHH, HHHT, HHTH, HTHH, and THHH.

   c. The probability of obtaining at least three heads is $\frac{5}{16}$ or 0.3125 or 31.25%.

2. a. Textbook questions 4, 5, and 6 of "Practice," p. 371

4.

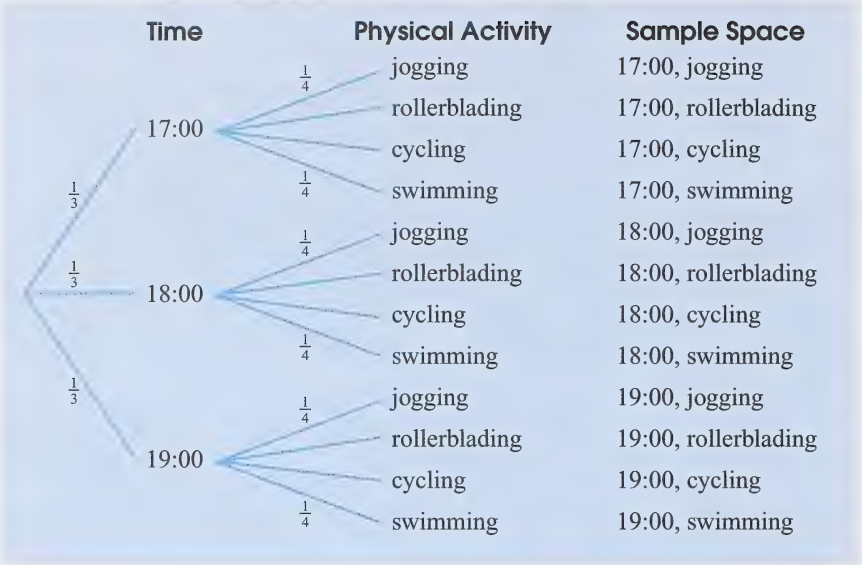


The probability of choosing a comedy with chips is

$$P(\text{comedy, chips}) = P(\text{comedy}) \times P(\text{chips})$$
$$= \frac{1}{4} \times \frac{1}{3}$$
$$= \frac{1}{12}.$$

Section 1: Activity 1 (continued)

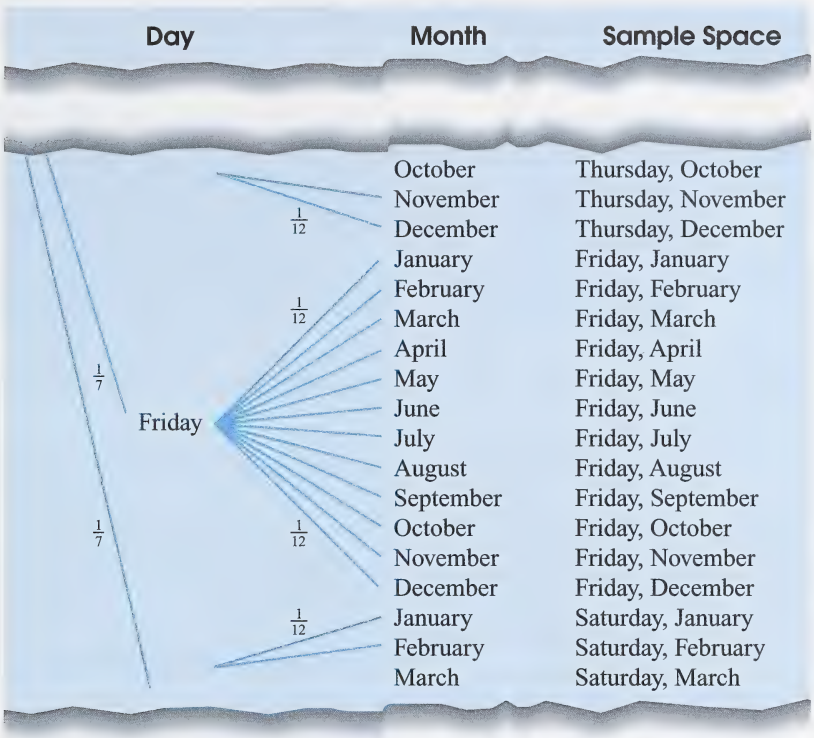
5.



The probability of choosing swimming at 17:00 is

$$P(\text{17:00, swimming}) = P(\text{17:00}) \times P(\text{swimming})$$
$$= \frac{1}{3} \times \frac{1}{4}$$
$$= \frac{1}{12}$$

6. To save space, only part of this probability tree diagram is shown.



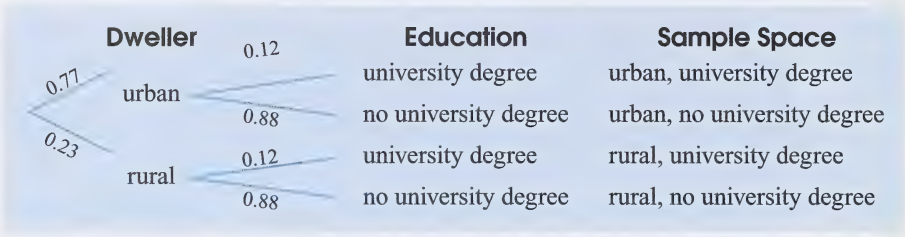
The probability of choosing Friday and April is

$$\begin{aligned} P(\text{Friday, April}) &= P(\text{Friday}) \times P(\text{April}) \\ &= \frac{1}{7} \times \frac{1}{12} \\ &= \frac{1}{84} \end{aligned}$$

Section 1: Activity 1 (continued)

b. Textbook questions 9, 10, and 14.a. to 14.e. of "Applications and Problem Solving," pp. 371 and 372

9. a.



The probability of selecting an urban dweller with a university degree is

$$\begin{aligned} P &= P(\text{urban}) \times P(\text{degree}) \\ &= 0.77 \times 0.12 \\ &= 0.0924 \text{ or } 9.24\% \end{aligned}$$

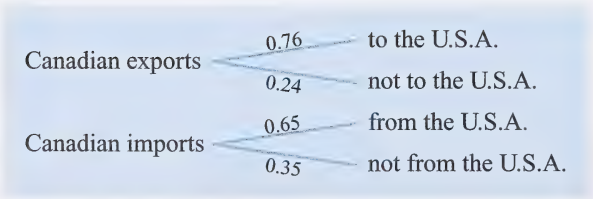
b. The probability of selecting a rural dweller with a university degree is

$$\begin{aligned} P &= P(\text{rural}) \times P(\text{degree}) \\ &= 0.23 \times 0.12 \\ &= 0.0276 \text{ or } 2.76\% \end{aligned}$$

c. The probability of selecting an urban dweller without a university degree is

$$\begin{aligned} P &= P(\text{urban}) \times P(\text{no degree}) \\ &= 0.77 \times 0.88 \\ &= 0.6776 \text{ or } 67.76\% \end{aligned}$$

10. a.



The probability that both trades are with the U.S.A. is

$$\begin{aligned} P &= P(\text{export to U.S.A.}) \times P(\text{import from U.S.A.}) \\ &= 0.76 \times 0.65 \\ &= 0.494 \text{ or } 49.4\% \end{aligned}$$

b. The probability that neither trade is with the U.S.A is

$$\begin{aligned} P &= P(\text{export not to U.S.A.}) \times P(\text{import not from U.S.A.}) \\ &= 0.24 \times 0.35 \\ &= 0.084 \text{ or } 8.4\% \end{aligned}$$

c. The probability that the export is to the U.S.A but the import is not from the U.S.A. is

$$\begin{aligned} P &= P(\text{export to U.S.A.}) \times P(\text{import not from U.S.A.}) \\ &= 0.76 \times 0.35 \\ &= 0.266 \text{ or } 26.6\% \end{aligned}$$

14. a. $$\begin{aligned} P(\text{no one scores}) &= P(\text{Orcas don't score}) \times P(\text{Bisons don't score}) \\ &= 0.65 \times 0.6 \\ &= 0.39 \end{aligned}$$

The probability that neither team scores in a round is 0.39.

b. $$\begin{aligned} P(\text{both teams score}) &= P(\text{Orcas score}) \times P(\text{Bisons score}) \\ &= 0.35 \times 0.40 \\ &= 0.14 \end{aligned}$$

The probability both teams score is 0.14.

c. If another round is necessary, it is because both teams scored or neither team scored.

$$\begin{aligned} \therefore P(\text{another round}) &= P(\text{both teams score}) + P(\text{neither team scores}) \\ &= 0.14 + 0.39 \\ &= 0.53 \end{aligned}$$

The probability that another round is necessary is 0.53.

d. For the Orcas to win on the second round, another round must have been required and the Bisons do not score in that round.

$$\begin{aligned} P(\text{Orcas win on 2nd round}) &= P(\text{another round is required}) \times P(\text{Orcas score}) \\ &\quad \times P(\text{Bisons don't score}) \\ &= 0.53 \times 0.35 \times 0.6 \\ &= 0.1113 \end{aligned}$$

The probability that the Orcas win on the second round is 0.1113.

Section 1: Activity 1 (continued)

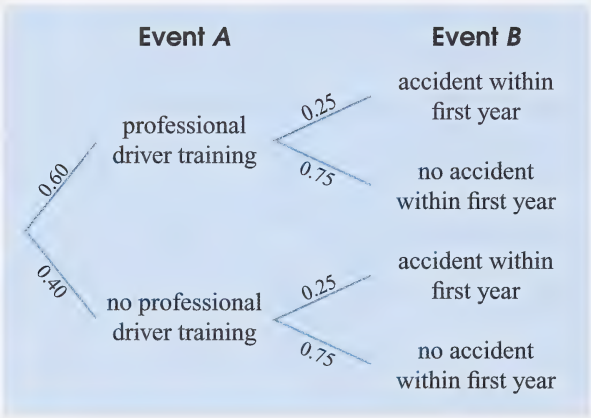
e. For the Bisons to win on the third round, two rounds must be completed and only the Bisons score in the third round.

$$\begin{aligned} P(\text{Bisons win on 3rd round}) &= P(\text{another round}) \times P(\text{another round}) \times P(\text{Bisons score}) \\ &\quad \times P(\text{Orcas don't score}) \\ &= 0.53 \times 0.53 \times 0.4 \times 0.65 \\ &= 0.073\,034 \end{aligned}$$

The probability that the Bisons win on the third round is 0.073 034.

3. Textbook questions 14, 15, 16.b., 16.c., 16.d., and 17 of "Applications and Problem Solving," pp. 380 and 381

14. a.



b. The probability that someone who has taken driver training will have an accident in the first year is

$$\begin{aligned} P &= 0.60 \times 0.25 \\ &= 0.15 \text{ or } 15\% \end{aligned}$$

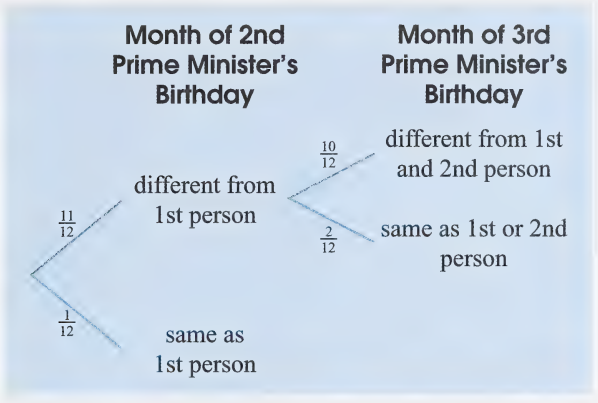
c. The result from the answer to question 14.b. does not support the studies because $15\% \neq 10\%$.

d. In question 14.a., the assumption made was that taking driver training and having an accident in the first year are independent. In fact, these events are dependent.

So, enrol in driver training to reduce your chances of having an accident.

15. a. Assume that each person's birthday is independent of each other. Find the probability of the complementary event: that no two of the next ten prime ministers will be born in the same month.

Draw a portion of the probability tree.



The probability that no two of the next ten prime ministers will be born in the same month is

$$P = \underbrace{\frac{11}{12} \times \frac{10}{12} \times \ldots \times \frac{3}{12}}_{9 \text{ factors}}$$
$$= \frac{{}_{11}P_9}{12^9}$$
$$\doteq 0.004$$

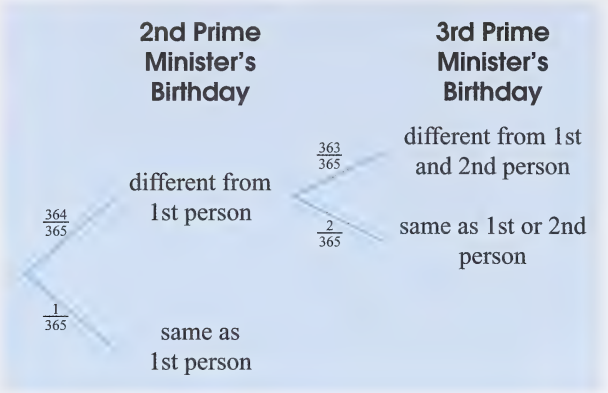
Therefore, the probability that at least two of the next ten prime ministers will be born in the same month is

$$P = 1 - P(\text{no 2 born in same month})$$
$$\doteq 1 - 0.004$$
$$\doteq 0.996 \text{ or } 99.6\%$$

Section 1: Activity 1 (continued)

b. Find the probability of the complementary event: that no two of the next ten prime ministers have the same birthday.

Draw a portion of the probability tree.



The probability that no two of the next ten prime ministers will have the same birthday is

$$P = \underbrace{\frac{364}{365} \times \frac{363}{365} \times \ldots \times \frac{356}{365}}_{9 \text{ factors}}$$

$$= \frac{{}_{364}P_9}{365^9}$$

$$\doteq 0.883$$

Therefore, the probability that at least two of the next ten prime ministers will have the same birthday is

$$P = 1 - P(\text{no 2 having same birthday})$$

$$\doteq 1 - 0.883$$

$$\doteq 0.117 \text{ or } 11.7\%$$

c. Find the probability of the complimentary event: that no 2 of the next 20 prime ministers will have the same birthday.

$$P = \underbrace{\frac{364}{365} \times \frac{363}{365} \times \ldots \times \frac{346}{365}}_{19 \text{ factors}}$$

$$= \frac{{}_{364}P_{19}}{365^{19}}$$

$$\doteq 0.589$$

Therefore, the probability that at least 2 of the next 20 prime ministers will have the same birthday is

$$P = 1 - P(\text{no 2 having same birthday})$$

$$\doteq 1 - 0.589$$

$$\doteq 0.411 \text{ or } 41.1\%$$

16. b. $P(\text{both have type O}) = P(\text{1st person has type O}) \times P(\text{2nd person has type O})$

$$= 0.44 \times 0.44$$

$$= 0.1936$$

The probability that a couple both have type O blood is 19.36%.

c. $P(\text{not type AB blood}) = 1 - P(\text{type AB})$

$$= 1 - 0.04$$

$$= 0.96$$

$$P(\text{neither has type AB}) = P\left(\begin{array}{c}\text{1st person does not} \\ \text{have type AB}\end{array}\right) \times P\left(\begin{array}{c}\text{2nd person does not} \\ \text{have type AB}\end{array}\right)$$

$$= 0.96 \times 0.96$$

$$= 0.9216$$

The probability that neither person in a couple has type AB blood is approximately 92.16%.

d. You would not consider blood types independent for two people that are related biologically.

Section 1: Activity 1 (continued)

17. a. It is easier to find the complementary event: that no 2 people live on the same street in the sample of 20.

$$P(\text{no 2 live on same street}) = \underbrace{\frac{114}{115} \times \frac{113}{115} \times \ldots \times \frac{96}{115}}_{19 \text{ factors}}$$

$$= \frac{{}_{114}P_{19}}{115^{19}}$$

$$\doteq 0.173$$

$$\therefore P(\text{at least 2 live on same street}) = 1 - P(\text{no 2 live on same street})$$

$$\doteq 1 - 0.173$$

$$\doteq 0.827$$

The probability that at least 2 of the 20 people selected live on the same street is approximately 82.7%.

b. The information "approximately the same number of residents" is relevant because it means living on each street is equally likely to be selected. If one of the streets had a lot more residents, it would be more likely than the others.

c. Once again, it is easier to use the complementary event: that no two people live on the same street in a random sample of n people.

$$P(\text{no 2 live on same street}) = \underbrace{\frac{114}{115} \times \frac{113}{115} \times \ldots \times \frac{114-n+1}{115}}_{n-1 \text{ factors}}$$

$$= \frac{{}_{114}P_{n-1}}{115^{n-1}}$$

$$\therefore P(\text{at least 2 live on same street}) = 1 - P(\text{no 2 live on same street})$$

$$0.75 \leq 1 - \frac{{}_{114}P_{n-1}}{115^{n-1}}$$

$$\frac{{}_{114}P_{n-1}}{115^{n-1}} \leq 0.25$$

There are two methods you can use to solve for n.

Method 1: Using Trial and Error

The value of n must be less than 20 because the probability that 2 of the 20 people live on the same street is approximately 83% (> 75%).

Try $n = 19$,

LS	RS
$\frac{{}_{114}P_{n-1}}{115^{n-1}}$	0.25
$= \frac{{}_{114}P_{19-1}}{115^{19-1}}$	
$= \frac{{}_{114}P_{18}}{115^{18}}$	
$\doteq 0.207$	

LS $\leq$ RS

Try $n = 18$,

LS	RS
$\frac{{}_{114}P_{n-1}}{115^{n-1}}$	0.25
$= \frac{{}_{114}P_{18-1}}{115^{18-1}}$	
$= \frac{{}_{114}P_{17}}{115^{17}}$	
$\doteq 0.246$	

LS $\leq$ RS

Try $n = 17$,

LS	RS
$\frac{{}_{114}P_{n-1}}{115^{n-1}}$	0.25
$= \frac{{}_{114}P_{17-1}}{115^{17-1}}$	
$= \frac{{}_{114}P_{16}}{115^{16}}$	
$\doteq 0.288$	

LS $\not\leq$ RS

Therefore, the canvasser must sample at least 18 people.

Section 1: Activity 1 (continued)

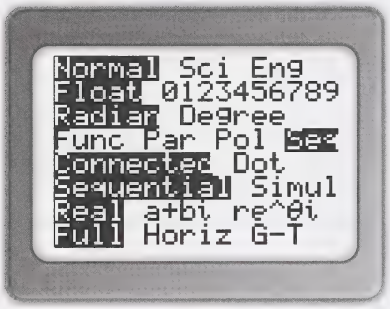
Method 2: Using a Table of Values

Use your graphing calculator to generate a table of values.

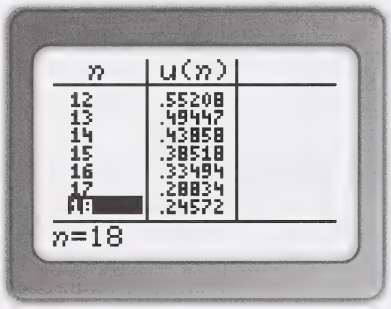
First, make sure your calculator is in Seq mode; then enter the following keystrokes.

Y= 1 1 4 MATH [Select the PRB menu.] 2 (X,T,Θ,n

− 1) ÷ 1 1 5 ^ (X,T,Θ,n − 1)



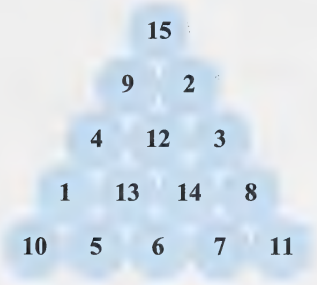
Press 2nd [TABLE] to see the table of values. Scroll to the value of n that results in $u(n) \leq 0.25$.

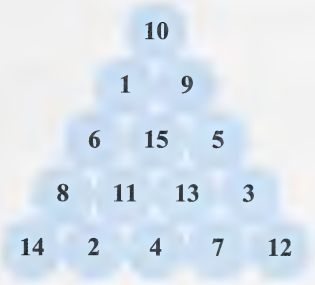


Therefore, the canvasser must sample at least 18 people.

4. Textbook question "NUMBER POWER," p. 381

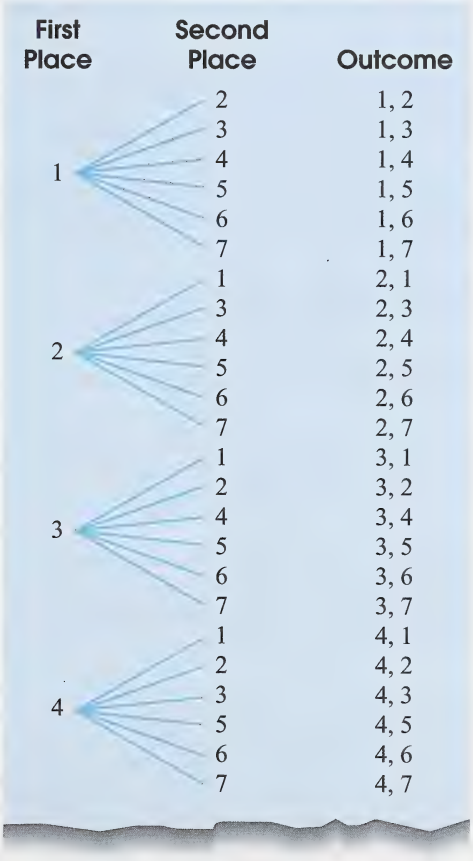
Answers may vary. Two sample answers are given.

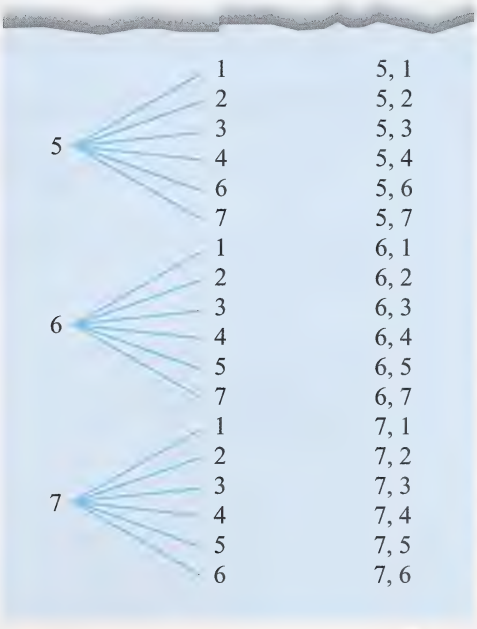




Section 1: Activity 2

1. a. Textbook question "Explore: Draw a Tree Diagram," p. 382





Section 1: Activity 2 (continued)

b. Textbook questions 1 to 3 of "Inquire," p. 382

1. There are 42 branches in the tree diagram.

2. Using the Fundamental Counting Principle, there are seven runners that could finish first, six runners that could finish second, and five runners that could finish third.

   $\therefore 7 \times 6 \times 5 = 210$

   If the tree diagram were to include the first, second, and third place finishers from the seven premier runners, 210 branches would be needed.

   If the tree diagram were to include the top ten finishers, 10! branches would be needed.

3. Assuming there are no ties, the number of different possible finishes of the n runners is $n!$.

   The number of possible top ten finishers is $n \times (n-1) \times (n-2) \times \ldots \times (n-9)$. This product can be written as ${}_nP_{10}$.

2. Textbook questions 1, 2, 4, and 5 of "Practice," p. 385

1. If exactly three females (F) must be on the committee, then the other two members are male (M).

   Because there are ten females; there are ${}_{10}C_3$ ways to choose the females. Because there are eight males, there are ${}_8C_2$ ways to choose the males. Therefore, the number of committees consisting of three females and two males is ${}_{10}C_3 \times {}_8C_2$.

   A committee of five people can be chosen a total of ${}_{18}C_5$ ways.

$$\therefore P(\text{exactly 3 F}) = \frac{\text{number of committees with 3 F and 2 M}}{\text{total number of committees}}$$
$$= \frac{{}_{10}C_3 \times {}_8C_2}{{}_{18}C_5}$$
$$= \frac{120 \times 28}{8568}$$
$$\doteq 0.392$$

   The probability that there are exactly three females on the committee is approximately 0.392 or 39.2%.

2. If exactly three males (M) must be on the committee, then the other two members must be female (F).

Because there are eight males to choose from, you can choose three males in ${}_8C_3$ ways. Because there are ten females to choose from, you can choose two females in ${}_{10}C_2$ ways.

There are ${}_{18}C_5$ ways to choose the committee.

$$\therefore P(\text{exactly 3 M}) = \frac{\text{number of committees with 3 M and 2 F}}{\text{total number of committees}}$$
$$= \frac{{}_8C_3 \times {}_{10}C_2}{{}_{18}C_5}$$
$$= \frac{56 \times 45}{8568}$$
$$\doteq 0.294 \text{ or } 29.4\%$$

4. There are 3! ways to arrange three people, and there is only 1 way to arrange the three people in descending order of age.

Therefore, the probability that three people line up in descending order of age is

$$P = \frac{\text{number of ways to arrange 3 people in descending order of age}}{\text{total number of ways to arrange 3 people}}$$
$$= \frac{1}{3!}$$
$$= \frac{1}{6}$$

5. The probability that three people line up in ascending order of age is the same as the probability of three people lining up in descending order of age. Therefore, the probability is $\frac{1}{6}$.

3. Textbook questions 11, 12, and 14 of "Applications and Problem Solving," p. 386

11. a. The coach differentiates among the positions of right wing, left wing, and centre on the forward line. Since players' positions are relevant, each assignment is a permutation. Therefore, the coach can assign positions for the first line in ${}_{15}P_3 = 2730$ ways.

Assume the second line is chosen after the first line. Since 3 people have already been assigned positions for the first line, there are 12 people possible for the second line. Therefore, the coach can assign the positions for the second line in ${}_{12}P_3 = 1320$ ways.

Section 1: Activity 2 (continued)

b. Method 1

Imagine there are 15 blanks, where each blank represents a position within the 5 lines.

Therefore, there are 15! ways of setting up the five lines.

The lines are chosen randomly; so, playing on the first line is no different than playing on any of the other lines. Therefore, the lines can be arranged in 5! ways.

$$\therefore \text{ Number of ways} = \frac{15!}{5!}$$
$$\doteq 1.09 \times 10^{10}$$

The coach can set up the five lines in approximately 1.09×10^{10} ways.

Method 2

The first line can be arranged in ${}_{15}P_3$ ways; the second line can be arranged in ${}_{12}P_3$ ways; The third line can be arranged in ${}_{9}P_3$ ways; the fourth line can be arranged in ${}_{6}P_3$ ways; and the fifth line can be arranged in ${}_{3}P_3$ ways.

The five lines can be arranged in 5! ways.

$$\therefore \text{ Number of ways} = \frac{{}_{15}P_3 \times {}_{12}P_3 \times {}_{9}P_3 \times {}_{6}P_3 \times {}_{3}P_3}{5!}$$
$$\doteq 1.09 \times 10^{10}$$

The coach can set up the five lines in approximately 1.09×10^{10} ways.

c. Method 1: Using Combinations and the Complementary Event

Because the only condition is that Zana and Kari play on the same line, the positions the girls play does not matter.

The complement of this event is that the girls do not play on the same line.

$\therefore$ P(girls play together) = 1 − P(girls don't play together)

To determine the probability that Zana and Kari do not play on the same line, start by placing Zana on the first line.

Since Kari cannot play on the same line as Zana, there are only 13 girls to choose from to play with Zana. Therefore, the number of ways the lines can be determined with Zana and Kari on different lines is as follows:

$$\text{Number of ways} = \frac{\overbrace{1}^{\text{1st line}} \times \overbrace{{}_{13}C_2}^{\text{2nd line}} \times \overbrace{{}_{12}C_3}^{\text{3rd line}} \times \overbrace{{}_{9}C_3}^{\text{4th line}} \times \overbrace{{}_{6}C_3}^{\text{5th line}} \times \overbrace{{}_{3}C_3}^{\text{5th line}}}{5!}$$

$$= \frac{{}_{13}C_2 \times {}_{12}C_3 \times {}_{9}C_3 \times {}_{6}C_3 \times {}_{3}C_3}{5!}$$

The total number of ways the lines can be chosen with Zana on the first line is as follows:

$$\text{Number of ways} = \frac{{}_{14}C_2 \times {}_{12}C_3 \times {}_{9}C_3 \times {}_{6}C_3 \times {}_{3}C_3}{5!}$$

The probability that the girls do not play on the same line is

$$P = \frac{\text{number of ways Zana and Kari don't play together}}{\text{total number of ways with Zana on the first line}}$$

$$= \frac{\frac{{}_{13}C_2 \times {}_{12}C_3 \times {}_{9}C_3 \times {}_{6}C_3 \times {}_{3}C_3}{5!}}{\frac{{}_{14}C_2 \times {}_{12}C_3 \times {}_{9}C_3 \times {}_{6}C_3 \times {}_{3}C_3}{5!}}$$

$$= \frac{{}_{13}C_2}{{}_{14}C_2}$$

$$= \frac{78}{91}$$

$$= \frac{6}{7}$$

Section 1: Activity 2 (continued)

Therefore, the probability that Zana and Kari play on the same line is

$$\begin{aligned} P &= 1 - P(\text{girls don't play together}) \\ &= 1 - \frac{6}{7} \\ &= \frac{1}{7} \end{aligned}$$

Method 2: Using Combinations

If Zana and Kari play together on the first line, 1 girl must be chosen from the remaining 13 to play with Zana and Kari. Therefore, the number of ways of setting up the five lines if Zana and Kari are on the first line is

$$\begin{aligned} \text{First line} &= 1 \times {}_{13}C_1 \times {}_{12}C_3 \times {}_9C_3 \times {}_6C_3 \times {}_3C_3 \\ &= 4\,804\,800 \end{aligned}$$

Similarly, the number of ways of setting up the five lines if Zana and Kari are on any of the other lines are as follows:

$$\begin{aligned} \text{Second line} &= {}_{13}C_3 \times 1 \times {}_{10}C_1 \times {}_9C_3 \times {}_6C_3 \times {}_3C_3 \\ &= 4\,804\,800 \end{aligned}$$

$$\begin{aligned} \text{Third line} &= {}_{13}C_3 \times {}_{10}C_3 \times 1 \times {}_7C_1 \times {}_6C_3 \times {}_3C_3 \\ &= 4\,804\,800 \end{aligned}$$

$$\begin{aligned} \text{Fourth line} &= {}_{13}C_3 \times {}_{10}C_3 \times {}_7C_3 \times 1 \times {}_4C_1 \times {}_3C_1 \\ &= 4\,804\,800 \end{aligned}$$

$$\begin{aligned} \text{Fifth line} &= {}_{13}C_3 \times {}_{10}C_3 \times {}_7C_3 \times {}_4C_3 \times 1 \times {}_1C_1 \\ &= 4\,804\,800 \end{aligned}$$

Therefore, the number of ways of setting up the five lines so that Zana and Kari play on the same is as follows:

$$\begin{aligned} \text{Number of ways} &= \frac{5(4\,804\,800)}{5!} \\ &= 200\,200 \end{aligned}$$

The total number of ways of setting up five lines is as follows:

$$\text{Number of ways} = \frac{{}_{15}C_3 \times {}_{12}C_3 \times {}_{9}C_3 \times {}_{6}C_3 \times {}_{3}C_3}{5!}$$
$$= 1\,401\,400$$

Therefore, the probability that Zana and Kari play on the same line is

$$P = \frac{\text{number of ways Zana and Kari play together}}{\text{total number of ways to set up 5 lines}}$$
$$= \frac{200\,200}{1\,401\,400}$$
$$= \frac{1}{7}$$

Method 3: Using Permutations

Determine the number of ways Zana and Kari could be positioned on a line. Let a blank represent the position that is open (left wing, centre, or right wing); let *Z* represent Zana; and let *K* represent Kari.

$Z\,K\,\_,\ Z\,\_\,K,\ K\,Z\,\_,\ \_\,Z\,K,\ K\,\_\,Z,\ \_\,K\,Z$

There are 6 ways Zana and Kari could line up on each line; there are 5 different lines Zana and Kari could play on; and there are 13! ways to arrange to other players.

Therefore, the number of ways of setting up five lines with Zana and Kari on the same line is as follows:

$$\text{Number of ways} = \frac{6 \times 5 \times 13!}{5!}$$

The total number of ways to set up the five lines is as follows:

$$\text{Number of ways} = \frac{15!}{5!}$$

Section 1: Activity 2 (continued)

Therefore, the probability that Zana and Kari play on the same line is

$$P = \frac{\text{number of ways Zana and Kari play together}}{\text{total number of ways to set up 5 lines}}$$

$$= \frac{\frac{6\times5\times13!}{3!}}{\frac{15!}{5!}}$$

$$= \frac{6\times5\times\overset{1}{\cancel{13!}}}{\underset{15\times14}{\cancel{15!}}}$$

$$= \frac{30}{210}$$

$$= \frac{1}{7}$$

d. If the coach does not differentiate between the three positions, it is a combination problem.

The coach can choose the first line in ${}_{15}C_3 = 455$ ways.

The coach can choose the second line in ${}_{12}C_3 = 220$ ways.

The coach can set up the five lines in ${}_{15}C_3 \times {}_{12}C_3 \times {}_9C_3 \times {}_6C_3 \times {}_3C_3$ ways; but because the order of the lines does not matter, the total is divided by 5!.

$$\therefore \text{ Total number of ways} = \frac{{}_{15}C_3 \times {}_{12}C_3 \times {}_9C_3 \times {}_6C_3 \times {}_3C_3}{5!}$$

$$= \frac{455\times220\times84\times20\times1}{120}$$

$$= 1\,401\,400$$

The coach can set up 1 401 400 lines if positions do not matter.

The probability that Zana and Kari will play on the same line will not change if the coach does not differentiate among the three positions on the line. The positions the girls play are irrelevant. Therefore, the probability Zana and Kari will play on the same line is still $\frac{1}{7}$.

The probability they are both chosen for a given line is

$$P = \frac{{}_2C_2 \times {}_{13}C_1}{{}_{15}C_3}$$
$$= \frac{1}{35}$$

Because there are five lines, the probability they will play together is $5 \times \frac{1}{35} = \frac{1}{7}$.

12. a. The order in which the cards are dealt to a person does not matter; so, the number of different hands is a combination. There is only 1 way to deal 13 spades to a specific player. Because there are 4 players, it is possible to deal a hand of all spades in 4 ways.

The number of 13-card hands is ${}_{52}C_{13}$.

$$\therefore P(\text{all spades}) = \frac{4}{{}_{52}C_{13}}$$
$$\doteq 6.3 \times 10^{-12}$$

b. There are four suits in a deck of cards; so, the probability of getting one suit is four times the probability of getting all spades.

$$P(\text{of only 1 suit}) = 4 \times \frac{4}{{}_{52}C_{13}}$$
$$= \frac{16}{{}_{52}C_{13}}$$
$$\doteq 2.5 \times 10^{-11}$$

c. The first player can be dealt ${}_{52}C_{13}$ different 13-card hands. From the remaining 39 cards (52 – 13), the second player can be dealt ${}_{39}C_{13}$ different hands. From the remaining 26 cards (39 – 13), the third player can be dealt ${}_{26}C_{13}$ different hands. The remaining 13 cards are dealt to the fourth player.

Therefore, the total number of ways the deck of cards can be dealt to four players is ${}_{52}C_{13} \times {}_{39}C_{13} \times {}_{26}C_{13} \times {}_{13}C_{13}$.

Assume the first player is dealt all clubs, the second player is dealt all diamonds, the third player is dealt all hearts, and the fourth player is dealt all spades. There is only 1 way to deal these players these suits. But because the suit the player receives does not matter, there are 4! ways each player can receive a complete suit.

Section 1: Activity 2 (continued)

Therefore, the probability of the four players receiving a complete suit is

$$P = \frac{\text{number of ways of each player receiving a complete suit}}{\text{total number of ways the cards can be dealt}}$$
$$= \frac{4!}{{}_{52}C_{13} \times {}_{39}C_{13} \times {}_{26}C_{13} \times {}_{13}C_{13}}$$
$$\doteq 4.5 \times 10^{-28}$$

d. The total number of hands possible is ${}_{52}C_{13}$.

There is only one way to be dealt four aces. Therefore, there are $1 \times {}_{48}C_9$ different hands with four aces.

Because there are four players in the game, there are $4 \times {}_{48}C_9$ ways a player has four aces.

Therefore, the probability that a player has four aces is

$$P = \frac{\text{number of ways of getting 4 aces}}{\text{total number of hands possible}}$$
$$= \frac{4 \times {}_{48}C_9}{{}_{52}C_{13}}$$
$$\doteq 0.011 \text{ or } 1.1\%$$

e. The total number of hands possible is ${}_{52}C_{13}$.

The number of hands without an ace or a face card is ${}_{36}C_{13}$.

Because there are four players, there are $4 \times {}_{36}C_{13}$ ways a player will not be dealt any aces or face cards.

Therefore, the probability that a player will not be dealt a face card or an ace is

$$P = \frac{\text{number of ways of getting no face cards or aces}}{\text{total number of hands possible}}$$
$$= \frac{4 \times {}_{36}C_{13}}{{}_{52}C_{13}}$$
$$\doteq 0.015 \text{ or } 1.5\%$$

14. a. The probability of rolling a 6 with one die is $\frac{1}{6}$. Using the Fundamental Counting Principle, the probability of rolling a 6 on all five dice is

$$\begin{aligned} P &= \frac{1}{6} \times \frac{1}{6} \times \frac{1}{6} \times \frac{1}{6} \times \frac{1}{6} \\ &= \frac{1}{6^5} \\ &= \frac{1}{7776} \end{aligned}$$

b. $$\begin{aligned} P(\text{any yahtzee}) &= 6 \times P(\text{yahtzee of 6s}) \\ &= 6 \times \frac{1}{6^5} \\ &= \frac{1}{6^4} \\ &= \frac{1}{1296} \end{aligned}$$

c. The sample space when all dice are different is as follows:

$$\{1, 2, 3, 4, 5\}, \{1, 2, 3, 4, 6\}, \{1, 2, 3, 5, 6\}, \{1, 2, 4, 5, 6\}, \{1, 3, 4, 5, 6\}, \{2, 3, 4, 5, 6\}$$

For each sample space, the numbers can be arranged in 5! ways. Therefore, the number of ways or rolling five different numbers is $6 \times 5!$.

Therefore, the probability of rolling not even one pair is

$$\begin{aligned} P &= \frac{\text{number of ways of rolling 5 different numbers}}{\text{total number of ways of rolling 5 dice}} \\ &= \frac{6 \times 5!}{6^5} \\ &= \frac{5!}{6^4} \\ &\doteq 0.093 \text{ or } 9.3\% \end{aligned}$$

Section 1: Activity 2 (continued)

4. Textbook question "LOGIC POWER," p. 386

Let g represent the number of green marbles, and let w represent the number of white marbles.

If two marbles are removed without replacements,

$$P(\text{both green}) = \frac{g}{g+w} \times \frac{g-1}{g+w-1}$$

$$\frac{2}{15} = \frac{g}{g+w} \times \frac{g-1}{g+w-1} \quad (1)$$

If three marbles are removed without replacements,

$$P(\text{all green}) = \frac{g}{g+w} \times \frac{g-1}{g+w-1} \times \frac{g-2}{g+w-2}$$

$$\frac{1}{30} = \frac{g}{g+w} \times \frac{g-1}{g+w-1} \times \frac{g-2}{g+w-2} \quad (2)$$

Substitute $\frac{2}{15}$ for $\left(\frac{g}{g+w} \times \frac{g-1}{g+w-1}\right)$ into (2).

$$\frac{1}{30} = \frac{g}{g+w} \times \frac{g-1}{g+w-1} \times \frac{g-2}{g+w-2}$$

$$\frac{1}{30} = \frac{2}{15} \times \frac{g-2}{g+w-2}$$

$$\frac{\cancel{15}^{1}}{2}\left(\frac{1}{\cancel{30}_{2}}\right) = \frac{\cancel{15}^{1}}{\cancel{2}_{1}}\left(\frac{\cancel{2}^{1}}{\cancel{15}_{1}} \times \frac{g-2}{g+w-2}\right)$$

$$\frac{1}{4} = \frac{g-2}{g+w-2}$$

$$g+w-2 = 4g-8$$

$$w = 3g-6$$

Use guess and check to determine the number of green and white marbles.

Try $g = 3$.

$$\begin{aligned} w &= 3g - 6 \\ &= 3(3) - 6 \\ &= 9 - 6 \\ &= 3 \end{aligned}$$

Check

LS	RS
$\frac{2}{15}$	$\frac{g}{g+w} \times \frac{g-1}{g+w-1}$
	$= \frac{3}{3+3} \times \frac{3-1}{3+3-1}$
	$= \frac{3}{6} \times \frac{2}{5}$
	$= \frac{1}{2} \times \frac{2}{5}$
	$= \frac{1}{5}$

LS $\neq$ RS

Try $g = 4$.

$$\begin{aligned} w &= 3g - 6 \\ &= 3(4) - 6 \\ &= 12 - 6 \\ &= 6 \end{aligned}$$

Check

LS	RS
$\frac{2}{15}$	$\frac{g}{g+w} \times \frac{g-1}{g+w-1}$
	$= \frac{4}{4+6} \times \frac{4-1}{4+6-1}$
	$= \frac{4}{10} \times \frac{3}{9}$
	$= \frac{2}{5} \times \frac{1}{3}$
	$= \frac{2}{15}$

LS = RS

LS	RS
$\frac{1}{30}$	$\frac{g}{g+w} \times \frac{g-1}{g+w-1} \times \frac{g-2}{g+w-2}$
	$= \frac{4}{4+6} \times \frac{4-1}{4+6-1} \times \frac{4-2}{4+6-2}$
	$= \frac{4}{10} \times \frac{3}{9} \times \frac{2}{8}$
	$= \frac{24}{720}$
	$= \frac{1}{30}$

LS = RS

There are 4 green marbles and 6 white marbles in the bag.

Section 1: Follow-up Activities

Extra Help

1. Textbook questions 3.b., 3.c., 4, 7, and 12.c. of "Review," pp. 396 and 397

3. b. Assume that Olivia's marks on her English essays are between 75% and 85% inclusive and that Andrew's marks on his English essays are between 70% and 78% inclusive.

It is possible for Olivia to get 11 different marks, and it is possible for Andrew to get 9 different marks. Using the Fundamental Counting Principle, there are $11 \times 9 = 99$ pairs of marks.

Let (a, b) represent a pair of marks, where a represents Olivia's mark and b represents Andrew's mark.

The sample space where Andrew scores higher than Olivia is as follows:

(75, 76), (75, 77), (75, 78), (76, 77), (76, 78), (77, 78)

There are 6 outcomes where Andrew scores higher.

$$\therefore P(\text{Andrew scores higher}) = \frac{6}{99}$$
$$= \frac{2}{33}$$

c. The sample space where both students score the same mark is as follows:

(75, 75), (76, 76), (77, 77), (78, 78)

$$\therefore P(\text{Olivia and Andrew receive the same mark}) = \frac{4}{99}$$

4. a.
$$P\begin{pmatrix}\text{Chi and Farhana}\\ \text{will go to U.B.C.}\end{pmatrix} = P\begin{pmatrix}\text{Chi will go}\\ \text{to U.B.C.}\end{pmatrix} \times P\begin{pmatrix}\text{Farhana will}\\ \text{go to U.B.C.}\end{pmatrix}$$
$$= 0.2 \times 0.25$$
$$= 0.05$$

b.
$$P\begin{pmatrix}\text{Chi will not go}\\ \text{to university}\end{pmatrix} = 1 - \left[P\begin{pmatrix}\text{Chi will go}\\ \text{to U.B.C.}\end{pmatrix} + P\begin{pmatrix}\text{Chi will go to}\\ \text{another university}\end{pmatrix}\right]$$
$$= 1 - [0.2 + 0.35]$$
$$= 1 - 0.55$$
$$= 0.45$$

c. $P(\text{Farhana does not go to U.B.C.}) = 1 - 0.25$
$= 0.75$

$P(\text{Chi does not go to U.B.C.}) = 1 - 0.2$
$= 0.8$

$$\therefore P\begin{pmatrix}\text{Chi or Farhana will go}\\ \text{to U.B.C., but not both}\end{pmatrix} = P\begin{pmatrix}\text{Chi goes to U.B.C. and Farhana}\\ \text{does not go to U.B.C.}\end{pmatrix} + P\begin{pmatrix}\text{Chi does not go}\\ \text{and Farhana does}\end{pmatrix}$$
$$= 0.2 \times 0.75 + 0.8 \times 0.25$$
$$= 0.15 + 0.2$$
$$= 0.35$$

7. a. The number of ways one teacher can be chosen is ${}_4C_1$.

The committee has to consist of five people; so, four students are needed. There is only one way to choose all the students.

The number of committees consisting of one teacher and four students is ${}_4C_1 \times 1$.

The total number of committees consisting of five people is ${}_8C_5$.

The probability that there is only one teacher on the committee is

$$P = \frac{{}_4C_1 \times 1}{{}_8C_5}$$
$$= \frac{4}{56}$$
$$= \frac{1}{14}$$

b. In order for the students to outnumber the teachers, there must be three students and two teachers or four students and one teacher.

The number of committees with three students and two teachers is ${}_4C_3 \times {}_4C_2$.

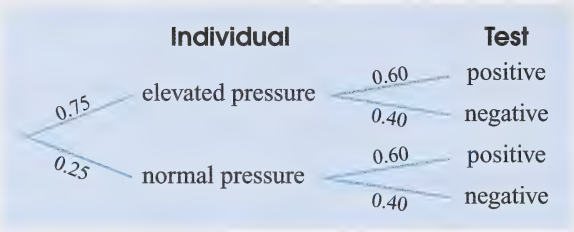
The number of committees with four students and one teacher is ${}_4C_4 \times {}_4C_1$.

The probability that the students outnumber the teachers is

$$P = \frac{\begin{pmatrix}\text{number of committees with}\\ \text{3 students and 2 teachers}\end{pmatrix} + \begin{pmatrix}\text{number of committees with}\\ \text{4 students and 1 teacher}\end{pmatrix}}{(\text{number of committees with 5 people})}$$
$$= \frac{{}_4C_3 \times {}_4C_2 + {}_4C_4 \times {}_4C_1}{{}_8C_5}$$
$$= \frac{24 + 4}{56}$$
$$= \frac{28}{56}$$
$$= \frac{1}{2}$$

Extra Help (continued)

c. The sample only involves glaucoma patients; so, a new probability tree is drawn.



$$\therefore P(\text{elevated pressure and positive test}) = (0.75)(0.6)$$
$$= 0.45 \text{ or } 45\%$$

The probability that a person with glaucoma actually tests positive is 45%.

2. Textbook question 6 of "Chapter Check," p. 398

6. a. There is only one correct password.

The total number of passwords with four characters and no repetitions is $26 \times 25 \times 24 \times 23$ or ${}_{26}P_4$.

Therefore, the probability that a hacker will guess the correct password on the first attempt is

$$P = \frac{\text{number of correct passwords}}{\text{total number of possible passwords}}$$
$$= \frac{1}{{}_{26}P_4}$$
$$= \frac{1}{358\,800}$$

b. The probability that a hacker will guess the password within three attempts is

$$P = 3 \times \frac{1}{358\,800}$$
$$= \frac{1}{119\,600}$$

c. The number of 8-character passwords using 68 characters with repetitions is

$$68 \times 68 \times 68 \times 68 \times 68 \times 68 \times 68 \times 68 = 68^8$$

Therefore, the probability the hacker will guess the password on the first attempt is

$$P = 1 \times \frac{\text{number of correct passwords}}{\text{maximum number of passwords}}$$
$$= \frac{1}{68^8}$$

Enrichment

Textbook questions 1 and 2 of "Exploring Math: The Pigeonhole Principle," p. 397

1. Let the month of the year represent the boxes. Therefore, there will be 12 boxes.

If at least 4 names are not in the same box when 36 names have been put in the boxes, then there must be 3 names in each box.

Now, when the 37th name is placed into a box, there will be 4 names in that box. Therefore, there are at least 4 people born in the same month.

2. Let each pair of socks represent a box. Therefore, there are 10 boxes to choose from.

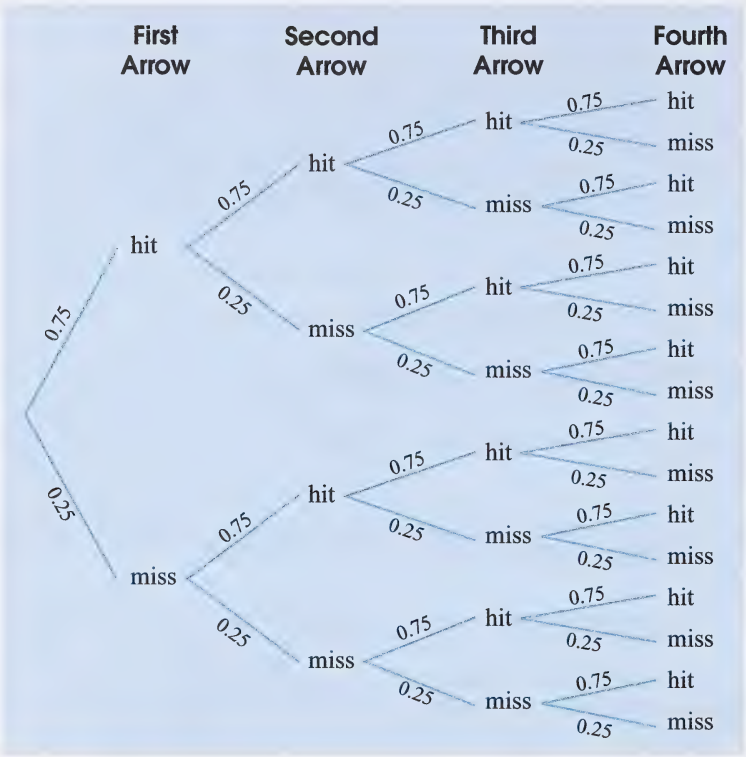
The worst-case scenario would be to pull out 10 different socks with no matches. Therefore, there would be 1 sock in each box. So, when the next sock is pulled out, you would have a match.

Therefore, to guarantee at least one match, 11 socks would have to be taken out.

Section 2: Activity 1

1. a. Textbook questions a. to d. of "Explore: Look for a Pattern," p. 404

a.



b.

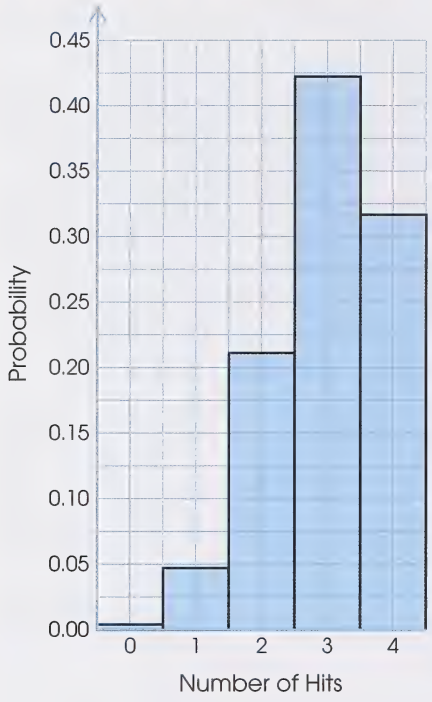
Arrow Outcomes	Hits	Probability
hit, hit, hit, hit	4	$0.75 \times 0.75 \times 0.75 \times 0.75 = (0.75)^4$
hit, hit, hit, miss	3	$0.75 \times 0.75 \times 0.75 \times 0.25 = (0.75)^3 (0.25)$
hit, hit, miss, hit	3	$0.75 \times 0.75 \times 0.25 \times 0.75 = (0.75)^3 (0.25)$
hit, hit, miss, miss	2	$0.75 \times 0.75 \times 0.25 \times 0.25 = (0.75)^2 (0.25)^2$
hit, miss, hit, hit	3	$0.75 \times 0.25 \times 0.75 \times 0.75 = (0.75)^3 (0.25)$
hit, miss, hit, miss	2	$0.75 \times 0.25 \times 0.75 \times 0.25 = (0.75)^2 (0.25)^2$
hit, miss, miss, hit	2	$0.75 \times 0.25 \times 0.25 \times 0.75 = (0.75)^2 (0.25)^2$
hit, miss, miss, miss	1	$0.75 \times 0.25 \times 0.25 \times 0.25 = (0.75) (0.25)^3$
miss, hit, hit, hit	3	$0.25 \times 0.75 \times 0.75 \times 0.75 = (0.75)^3 (0.25)$
miss, hit, hit, miss	2	$0.25 \times 0.75 \times 0.75 \times 0.25 = (0.75)^2 (0.25)^2$
miss, hit, miss, hit	2	$0.25 \times 0.75 \times 0.25 \times 0.75 = (0.75)^2 (0.25)^2$
miss, hit, miss, miss	1	$0.25 \times 0.75 \times 0.25 \times 0.25 = (0.75) (0.25)^3$
miss, miss, hit, hit	2	$0.25 \times 0.25 \times 0.75 \times 0.75 = (0.75)^2 (0.25)^2$
miss, miss, hit, miss	1	$0.25 \times 0.25 \times 0.75 \times 0.25 = (0.75) (0.25)^3$
miss, miss, miss, hit	1	$0.25 \times 0.25 \times 0.25 \times 0.75 = (0.75) (0.25)^3$
miss, miss, miss, miss	0	$0.25 \times 0.25 \times 0.25 \times 0.25 = (0.25)^4$

c.

Hits in 4 Throws	Ways	Probability of Each	Total Probability
4	1	$(0.75)^4$	$(0.75)^4$
3	4	$(0.75)^3(0.25)$	$4(0.75)^3(0.25)$
2	6	$(0.75)^2(0.25)^2$	$6(0.75)^2(0.25)^2$
1	4	$(0.75)(0.25)^3$	$4(0.75)(0.25)^3$
0	1	$(0.25)^4$	$(0.25)^4$

Section 2: Activity 1 (continued)

d.



b. Textbook questions 1 to 5 of "Inquire," p. 404

1. The exponents in the calculation of the probability of the number of successes (hits) always add up to 4, which is the number of trials.

2. The factors are $(0.75)^h$ and $(0.25)^m$, where h is the number of hits and m is the number of misses. For each term, $h + m = 4$. Also, the coefficient of each term is ${}_4C_h$.

3. $(H+M)^4 = {}_4C_0(H)^{4-0}(M)^0 + {}_4C_1(H)^{4-1}(M)^1 + {}_4C_2(H)^{4-2}(M)^2 + {}_4C_3(H)^{4-3}(M)^3 + {}_4C_4(H)^{4-4}(M)^4$

$= {}_4C_0H^4 + {}_4C_1H^3M + {}_4C_2H^2M^2 + {}_4C_3HM^3 + {}_4C_4M^4$

$= H^4 + 4H^3M + 6H^2M^2 + 4HM^3 + M^4$

Substitute H = 0.75 and M = 0.25.

$(H+M)^4 = H^4 + 4H^3M + 6H^2M^2 + 4HM^3 + M^4$

$(0.75+0.25)^4 = (0.75)^4 + 4(0.75)^3(0.25) + 6(0.75)^2(0.25)^2 + 4(0.75)(0.25)^3 + (0.25)^4$

The terms are the same values as those found in part c. of Explore.

Because Kaia can hit the bull's-eye 0, 1, 2, 3, or 4 times, this polynomial includes all the probabilities of possible outcomes for Kaia.

4. The polynomial that should be considered if Kaia shoots three arrows is $(0.75 + 0.25)^3$.

5. The bar of the histogram corresponding to 0 hits is very small. The tallest bar, which means the most likely to occur, corresponds to 3 hits.

The total area of all the bars is 1, because the height of each bar is the probability and the width is 1. The area of each bar is, therefore, the probability. Since all possible outcomes are represented, the probability is 100% or 1.

2. a. Textbook questions 3, 7, 8, 9, and 10 of "Practice," p. 407

3. $n = 20$ (number of trials)

$x = 10$ (number of successes)

$p = 0.5$ (probability of success on each trial)

$q = 1 - 0.5 = 0.5$ (probability of failure on each trial)

$P(10 \text{ successes}) = {}_nC_x p^x q^{n-x}$

$= {}_{20}C_{10}(0.5)^{10}(0.5)^{20-10}$

$= {}_{20}C_{10}(0.5)^{10}(0.5)^{10}$

$= 184\,756(0.5)^{20}$

$\doteq 0.176$ or 17.6%

Recall: ${}_{20}C_{10}$ can be calculated using the following keystrokes.

2 0 MATH [Select the PRB menu.]

3 (3:nCr) 1 0 ENTER

The probability of ten successes is approximately 0.176 or 17.6%.

Section 2: Activity 1 (continued)

7. $n = 15$ (number of trials)

$x = 14$ (number of successes)

$p = 0.99$ (probability of success on each trial)

$q = 1 - 0.99 = 0.01$ (probability of failure on each trial)

$$\begin{aligned} P(1 \text{ failure}) &= {}_nC_x p^x q^{n-x} \\ &= {}_{15}C_{14}(0.99)^{14}(0.01)^{15-14} \\ &= 15(0.99)^{14}(0.01)^1 \\ &\doteq 0.130 \text{ or } 13.0\% \end{aligned}$$

The probability of one failure is approximately 0.130 or 13.0%.

8. At least three successes means $x = 3$, $x = 4$, $x = 5$, or $x = 6$.

$\therefore\ n = 6$

$x = 3, 4, 5, 6$

$p = 0.35$

$q = 1 - 0.35 = 0.65$

Method 1

$$\begin{aligned} P(\text{at least 3 successes}) &= P(3 \text{ successes}) + P(4 \text{ successes}) + P(5 \text{ successes}) + P(6 \text{ successes}) \\ &= {}_6C_3(0.35)^3(0.65)^3 + {}_6C_4(0.35)^4(0.65)^2 + {}_6C_5(0.35)^5(0.65)^1 \\ &\quad + {}_6C_6(0.35)^6(0.65)^0 \\ &\doteq 0.353 \end{aligned}$$

The probability of at least three successes is approximately 0.353 or 35.3%.

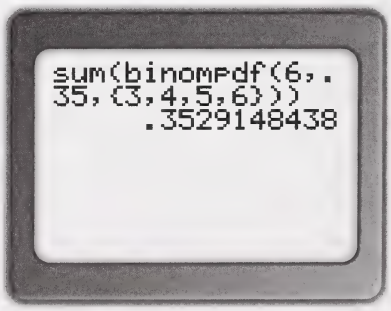
Method 2

[2nd] [LIST] [Select the MATH menu.] [5] (5:sum()

[2nd] [DISTR] [0] (0:binompdf() [6] [,] [•]

[3] [5] [,] [2nd] [{] [3] [,] [4] [,] [5]

[,] [6] [2nd] [}] [)] [)] [ENTER]



The probability of at least three successes is approximately 0.353 or 35.3%.

9. No more than three successes means $x = 0$, $x = 1$, $x = 2$, or $x = 3$.

$\therefore\ n = 100$
$x = 0, 1, 2, 3$
$p = 0.01$
$q = 1 - 0.01 = 0.99$

Method 1

$$P\left(\begin{matrix}\text{no more than}\\ \text{3 successes}\end{matrix}\right) = P(0 \text{ successes}) + P(1 \text{ success}) + P(2 \text{ successes}) + P(3 \text{ successes})$$
$$= {}_{100}C_0\,(0.01)^0\,(0.99)^{100} + {}_{100}C_1\,(0.01)^1\,(0.99)^{99} + {}_{100}C_2\,(0.01)^2\,(0.99)^{98}$$
$$+ {}_{100}C_3\,(0.01)^3\,(0.99)^{97}$$
$$\doteq 0.982$$

The probability of no more than three successes is approximately 0.982 or 98.2%.

Method 2

2nd [LIST] [Select the MATH menu.] 5 (5:sum() 2nd [DISTR] 0 (0:binompdf()
1 0 0 , • 0 1 , 2nd [{] 0 , 1 , 2 , 3
2nd [}])) ENTER

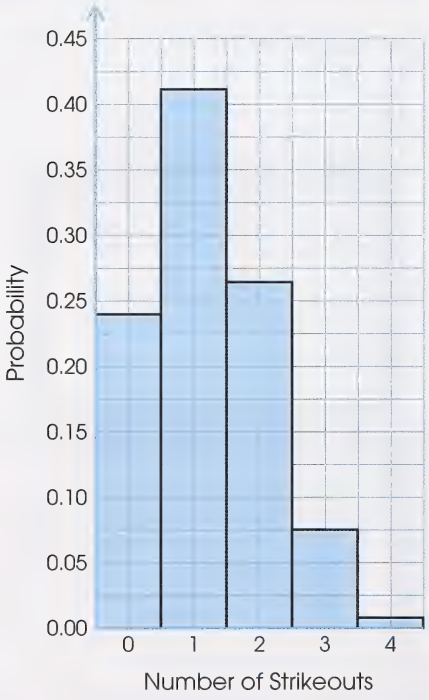
The probability of no more than three successes is approximately 0.982 or 98.2%.

Section 2: Activity 1 (continued)

10. Find all possible outcomes and their probabilities.

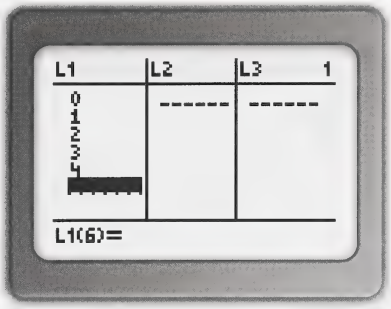
$n = 4$ (the number of at-bats)
$x = 0, 1, 2, 3, 4$ (the possible number of times a slugger strikes out)
$p = 0.3$ (the chance of success or striking out)
$q = 1 - 0.3 = 0.7$ (the chance of failure or hitting the ball)

Number of Strikeouts	Probability
0	${}_4C_0\,(0.3)^0\,(0.7)^4 = 0.2401$
1	${}_4C_1\,(0.3)^1\,(0.7)^3 = 0.4116$
2	${}_4C_2\,(0.3)^2\,(0.7)^2 = 0.2646$
3	${}_4C_3\,(0.3)^3\,(0.7)^1 = 0.0756$
4	${}_4C_4\,(0.3)^4\,(0.7)^0 = 0.0081$



The histogram can be drawn on your graphing calculator.

First, enter the values of x (0, 1, 2, 3, and 4) in list L1.

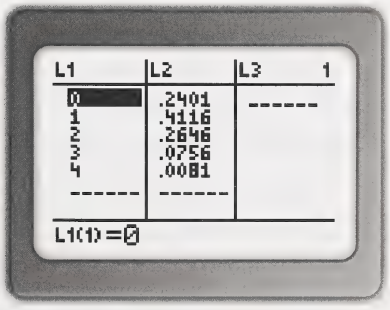


Now, enter the data from the probability distribution as list L2.

2nd [QUIT] 2nd [DISTR] 0 (0:binompdf() 4 , • 3) STO▸

2nd [L2] ENTER

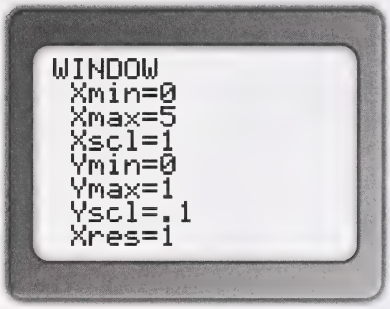
Press STAT 1 to view the lists.



Section 2: Activity 1 (continued)

To display a histogram, press [2nd] [STAT PLOT] [1]. Make sure your display matches the following display.

Using the window settings given, press [GRAPH] to view the histogram.



b. Textbook questions 14, 15, 17, 19, 21, and 22 of "Applications and Problem Solving," pp. 407 and 408

14. a. In this context, consider a success to be guessing the correct answer. A pass is considered to be 50% or higher, so Colin has to guess 5, 6, 7, 8, 9, or 10 questions correctly.

$\therefore n = 10$ (number of trials or questions on the quiz)

$x = 5, 6, 7, 8, 9, 10$ (number of questions Colin must get right to pass the quiz)

$p = 0.2$ (probability of success on each trial)

$q = 1 - 0.2 = 0.8$ (probability of failure on each trial)

$$\therefore P\left(\begin{matrix}\text{at least}\\ 5\text{ successes}\end{matrix}\right) = P(5\text{ successes}) + P(6\text{ successes}) + \ldots + P(10\text{ successes})$$
$$= {}_{10}C_5\,(0.2)^5\,(0.8)^5 + {}_{10}C_6\,(0.2)^6\,(0.8)^4 + {}_{10}C_7\,(0.2)^7\,(0.8)^3$$
$$+ {}_{10}C_8\,(0.2)^8\,(0.8)^2 + {}_{10}C_9\,(0.2)^9\,(0.8)^1 + {}_{10}C_{10}\,(0.2)^{10}\,(0.8)^0$$

To determine the sum with a graphing calculator, press the following.

[2nd] [LIST] [Select the MATH menu.] [5] (5:sum() [2nd] [DISTR] [0]
(0:binompdf() [1] [0] [,] [•] [2] [,] [2nd] [{] [5] [,] [6] [,]
[7] [,] [8] [,] [9] [,] [1] [0] [2nd] [}] [)] [)] [ENTER]

If Colin guesses the answer to each question, the probability he will pass the test is approximately 0.033 or 3.3%.

b. Consider a success to be answering the question correctly. To pass the quiz, Diana will have to answer 5, 6, 7, 8, 9, or 10 questions correctly.

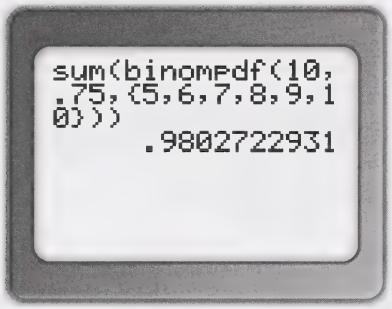
$\therefore n = 10$ (number of trials or questions on the quiz)
$x = 5, 6, 7, 8, 9, 10$
$p = 0.75$
$q = 1 - 0.75 = 0.25$

$$\therefore P\left(\begin{matrix}\text{at least}\\ 5\text{ successes}\end{matrix}\right) = P(5\text{ successes}) + P(6\text{ successes}) + \ldots + P(10\text{ successes})$$
$$= {}_{10}C_5\,(0.75)^5\,(0.25)^5 + {}_{10}C_6\,(0.75)^6\,(0.25)^4 + {}_{10}C_7\,(0.75)^7\,(0.25)^3$$
$$+ {}_{10}C_8\,(0.75)^8\,(0.25)^2 + {}_{10}C_9\,(0.75)^9\,(0.25)^1$$
$$+ {}_{10}C_{10}\,(0.75)^{10}\,(0.25)^0$$

Section 2: Activity 1 (continued)

To determine the sum with the calculator, press the following.

2nd [LIST] [Select the MATH menu.] 5 2nd [DISTR] 0 1 0 ,
• 7 5 , 2nd [{] 5 , 6 , 7 , 8 , 9 ,
1 0 2nd [}])) ENTER



The probability Diana will pass the quiz if she has a 75% chance of answering each question correctly is approximately 0.980 or 98.0%.

15. a. A success is a product that is defect-free.

$\therefore\ n = 50$ (number of components ordered)
$x = 50$ (number of components defect-free)
$p = 0.99$
$q = 0.01$

$$P(50 \text{ successes}) = {}_{50}C_{50}\,(0.99)^{50}\,(0.01)^{0}$$
$$\doteq 0.605 \text{ or } 60.5\%$$

The probability that none of the components in the order are defective is approximately 0.605 or 60.5%.

b. $n = 50$ (number of components ordered)
$x = 0, 1, 2, \ldots, 49$ (at least one defective means the number of successes is $0, 1, \ldots, 49$)
$p = 0.99$
$q = 0.01$

$$\begin{aligned}\therefore P(\text{at least one defective}) &= P(0 \text{ successes}) + P(1 \text{ success}) + \ldots + P(49 \text{ successes}) \\ &= 1 - P(50 \text{ successes}) \\ &\doteq 1 - 0.605 \\ &\doteq 0.395\end{aligned}$$

The probability that there is at least one defective product in the order is 0.395 or 39.5%.

c. $n = 50$ (number of components ordered)
$x = 0, 1, \ldots, 48$ (at least two defective means the number of successes is $0, 1, \ldots, 48$)
$p = 0.99$
$q = 0.01$

$$\begin{aligned}\therefore P(\text{at least two defective}) &= P(0 \text{ successes}) + P(1 \text{ success}) + \ldots + P(48 \text{ successes}) \\ &= 1 - P(49 \text{ successes}) - P(50 \text{ successes}) \\ &= 1 - {}_{50}C_{49}(0.99)^{49}(0.01)^{1} - {}_{50}C_{50}(0.99)^{50}(0.01)^{0} \\ &\doteq 1 - 0.306 - 0.605 \\ &\doteq 0.089 \text{ or } 8.9\%\end{aligned}$$

The probability that there is at least two defective products in the order is 0.089 or 8.9%.

Section 2: Activity 1 (continued)

17. a. The probability of double sixes rolled with a pair of dice is $\frac{1}{36}$.

$\therefore\ n = 20$ (number of trials)

$x = 2$ (number of successes)

$p = \frac{1}{36}$ (probability of success on each trial)

$q = \frac{35}{36}$ (probability of failure on each trial)

$\therefore\ P(\text{rolling double sixes twice}) = P(2 \text{ successes})$

$$= {}_{20}C_2 \left(\frac{1}{36}\right)^2 \left(\frac{35}{36}\right)^{18}$$

$$\doteq 0.088 \text{ or } 8.8\%$$

The probability that double sixes are rolled exactly twice is approximately 0.088 or 8.8%.

b. $n = 20$

$x = 2, 3$

$p = \frac{1}{36}$

$q = \frac{35}{36}$

The probability that double sixes are rolled more than once, but no more than three times, is

$$P = P(2 \text{ successes}) + P(3 \text{ successes})$$

$$= {}_{20}C_2 \left(\frac{1}{36}\right)^2 \left(\frac{35}{36}\right)^{18} + {}_{20}C_3 \left(\frac{1}{36}\right)^3 \left(\frac{35}{36}\right)^{17}$$

$$\doteq 0.103 \text{ or } 10.3\%$$

19. a. $n = 12$ (number of trials or intersections)

$x = 0, 1, 2$ (number of times he can stop at intersections is fewer than 3)

$p = 0.5$ (probability of success or the probability he has to stop)

$q = 0.5$ (probability of failure or the probability he has a green light)

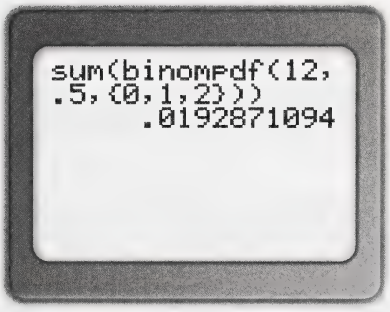
$$\therefore P\left(\begin{array}{c}\text{he is stopped at fewer} \\ \text{than 3 sets of lights}\end{array}\right) = P(0 \text{ successes}) + P(1 \text{ success}) + P(2 \text{ successes})$$

$$= {}_{12}C_0 (0.5)^0 (0.5)^{12} + {}_{12}C_1 (0.5)^1 (0.5)^{11} + {}_{12}C_2 (0.5)^2 (0.5)^{10}$$

To determine the sum with a calculator, press the following.

2nd [LIST] [Select the MATH menu.] 5 2nd [DISTR] 0 1 2 ,

• 5 , 2nd [{] 0 , 1 , 2 2nd [}])) ENTER



The probability Alan has time for coffee on any given day is approximately 0.019 or 1.9%.

Section 2: Activity 1 (continued)

b. First, determine the probability that Alan is late for work.

$\therefore\ n = 12$ (number of trials or intersections)

$x = 9, 10, 11, 12$ (number of times Alan is stopped at intersections)

$p = 0.5$

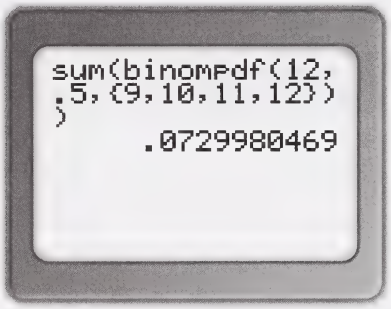
$q = 0.5$

$$P\left(\begin{array}{c}\text{Alan is stopped}\\ \text{more than 8 times}\end{array}\right) = P\left(\begin{array}{c}9\\ \text{successes}\end{array}\right) + P\left(\begin{array}{c}10\\ \text{successes}\end{array}\right) + P\left(\begin{array}{c}11\\ \text{successes}\end{array}\right) + P\left(\begin{array}{c}12\\ \text{successes}\end{array}\right)$$

$$= {}_{12}C_9\,(0.5)^9\,(0.5)^3 + {}_{12}C_{10}\,(0.5)^{10}\,(0.5)^2 + {}_{12}C_{11}\,(0.5)^{11}\,(0.5)^1 + {}_{12}C_{12}\,(0.5)^{12}\,(0.5)^0$$

To determine the sum with a calculator, press the following.

[2nd] [LIST] [Select the MATH menu.] [5] [2nd] [DISTR] [0] [1] [2] [,] [•] [5] [,] [2nd] [{] [9] [,] [1] [0] [,] [1] [1] [,] [1] [2] [2nd] [}] [)] [)] [ENTER]



The probability of Alan being late on a given day is approximately 0.073 or 7.3%.

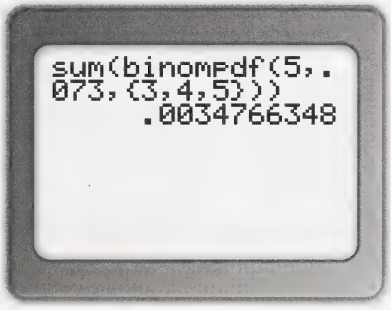
Now, Alan has to see the manager if he is late more than twice in a five-day work week.

$\therefore\ n = 5$ (number of trials or number of days in a week)
$x = 3, 4, 5$ (number of days Alan is late)
$p \doteq 0.073$ (probability of success or being late)
$q \doteq 0.927$ (probability of failure)

$$\therefore\ P(\text{Alan is late more than twice}) = P(3 \text{ lates}) + P(4 \text{ lates}) + P(5 \text{ lates})$$
$$\doteq {}_5C_3 (0.073)^3 (0.927)^2 + {}_5C_4 (0.073)^4 (0.927)^1 + {}_5C_5 (0.073)^5 (0.927)^0$$

To determine the sum with a calculator, press the following.

2nd [LIST] [Select the MATH menu.] 5 2nd [DISTR] 0 5 ,
• 0 7 3 , 2nd [{] 3 , 4 , 5 2nd [}])
) ENTER



The probability that Alan will have to see the manager is approximately 0.003 or 0.3%.

Section 2: Activity 1 (continued)

21. a. P(6 heads) = 0.26

The probability of seeing 6 heads is 0.26.

b. $P(\text{at least 4 heads}) = P(4 \text{ heads}) + P(5 \text{ heads}) + P(6 \text{ heads})$

$= 0.24 + 0.40 + 0.26$

$= 0.90$

The probability of seeing at least 4 heads is 0.90.

c. You need to determine the value of p.

$n = 6$ (number of trials or tosses of the coin)

$x = 6$

$p = ?$

$q = 1 - p$

Because $P(6 \text{ heads}) = 0.26$,

$${}_6C_6 p^6 (1-p)^0 = 0.26$$

$$p^6 = 0.26$$

$$p = (0.26)^{\frac{1}{6}}$$

$$\doteq 0.80$$

The probability of getting a head is approximately 0.80.

22. Lina's reasoning is not correct because you have to multiply the probabilities of making the shots. For example, the probability of making three shots is $0.85 \times 0.85 \times 0.85 = (0.85)^3$. This is not the same as 3×0.85.

In order for Carter to make 13 shots out of 15, it means he misses 2 shots. The probability, if order matters, would be $(0.85)^{13}(0.15)^2$.

The number of ways the factors 0.85 and 0.15 can be arranged is ${}_{15}C_{13}$ (describes where you would "place" the shots made) or ${}_{15}C_2$ (describes where you would "place" the shots missed).

$$\therefore P(\text{Carter makes exactly 13 baskets}) = {}_{15}C_{13}(0.85)^{13}(0.15)^2$$
$$\doteq 0.286$$

$$\therefore P(\text{Carter does not make exactly 13 baskets}) = 1 - P(\text{Carter makes exactly 13 baskets})$$
$$\doteq 1 - 0.286$$
$$\doteq 0.714$$

It is much more likely that Carter does not make exactly 13 baskets.

3. If you use 10 coins and compare the red histogram to the blue histogram, you will notice a marked difference between them. However, the two histograms will resemble each other more closely when the stop frequency is 100 and be almost identical when the stop frequency is 1000. As the stop frequency (number of trials) increases, a much smoother histogram is obtained. The shape of the curve is like a bell.

4. The blue histogram for $p = 0.5$ is symmetric, whereas the blue histogram for $p = 0.2$ is not symmetric. If $p = 0.5$, it is not likely to get 0 heads or 10 heads. If $p = 0.2$, it is very likely to get 0 heads. It is not likely to get 6, 7, 8, 9, or 10 heads.

Section 2: Activity 2

1. a. Textbook question "Explore: The Spread of Data," p. 409

The mean for the cartons are as follows:

Carton A: $\bar{x} = \dfrac{49+51+49+51+49+51+49+51+49+51}{10}$

$= \dfrac{500}{10}$

$= 50$

Carton B: $\bar{x} = \dfrac{48+52+48+52+48+52+48+52+48+52}{10}$

$= \dfrac{500}{10}$

$= 50$

Carton C: $\bar{x} = \dfrac{48+51+49+52+47+52+46+51+53+51}{10}$

$= \dfrac{500}{10}$

$= 50$

Carton D: $\bar{x} = \dfrac{54+49+46+49+51+53+50+50+49+49}{10}$

$= \dfrac{500}{10}$

$= 50$

Carton E: $\bar{x} = \dfrac{60+36+31+50+48+50+54+56+52+63}{10}$

$= \dfrac{500}{10}$

$= 50$

The range is the difference between the greatest and least data points. The range of the cartons are as follows:

Carton A: range = 51 – 49
= 2

Carton B: range = 52 – 48
= 4

Carton C: range = 53 – 46
= 7

Carton D: range = 54 – 46
= 8

Carton E: range = 63 – 31
= 32

b. Textbook questions 1 to 4 of "Inquire," p. 409

1. The data for all the cartons have the same number of data points and the same mean, but they have different ranges.

2. Carton E has the widest range of data, 32. Carton A has the least range of data, 2.

3. The cartons ranked from least to greatest according to their maximum deviation is Carton A, Carton B, Carton C, Carton D, and Carton E.

4. Answers will vary. A sample answer is given.

   A list where the mean is 50 and the range is 20 is as follows:

   Carton G: 40, 41, 42, 43, 44, 56, 57, 58, 59, 60

2. Textbook questions 1 and 3.a. of "Practice," p. 412

1. **a.** The range of each set of data is 100.

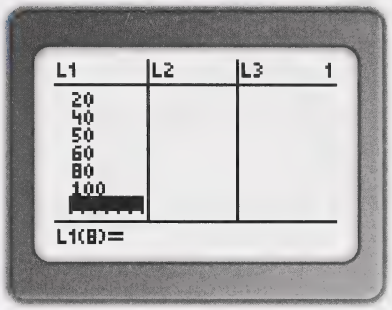
   Set C has values furthest from the mean; so, it has the greatest standard deviation. Set B has values closest to the mean; so, it has the least standard deviation.

Section 2: Activity 2 (continued)

b. Set A

Enter the data as a list.

STAT 1 (1:Edit…) 0 ENTER 2 0 ENTER 4 0 ENTER 5 0 ENTER 6 0 ENTER 8 0 ENTER 1 0 0 ENTER



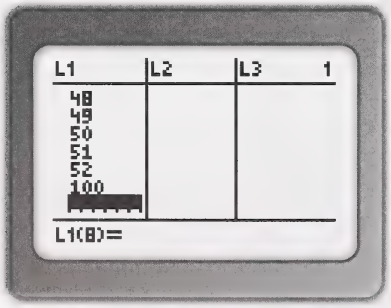
Next, press the following.

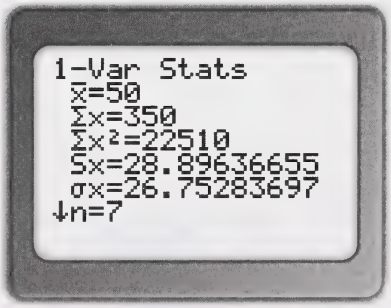
2nd [QUIT] STAT [Select the CALC menu.] 1 (1:1-Var Stats) ENTER

The standard deviation for set A is approximately 31.62.

Set B

Enter the data as a list to determine the standard deviation.

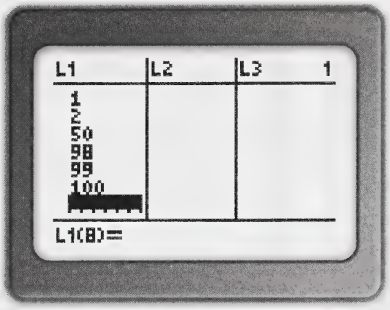




The standard deviation for set B is approximately 26.75.

Set C

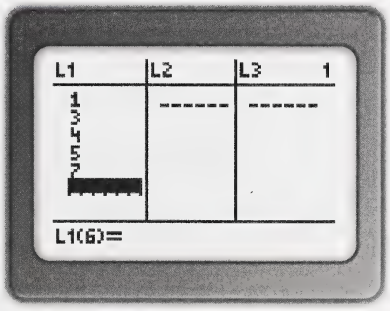
Enter the terms as a list to determine the standard deviation.



The standard deviation for set C is approximately 45.37.

The predictions made in question 1.a. were correct!

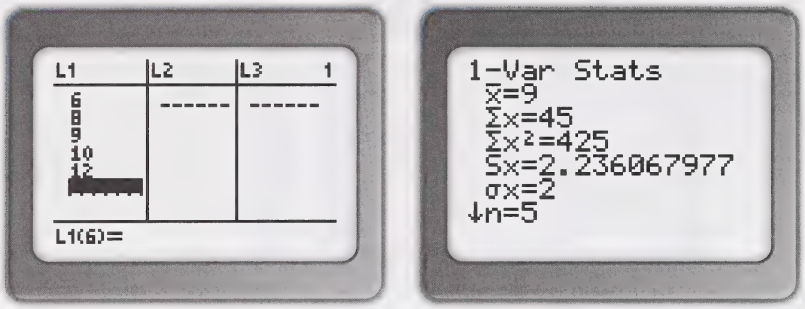
3. a. Set A



The mean is 4, and the standard deviation is 2.

Section 2: Activity 2 (continued)

Set B



The mean is 9, and the standard deviation is 2.

The standard deviations of both sets are the same. Also, each element in set B is 5 more than the corresponding element in set A.

3. a. Textbook question 4 of "Practice," p. 413

4. a. $\bar{x} = 355$ and $\sigma = 1.5$

If $x = 354$, then

$$\begin{aligned} z &= \frac{x - \bar{x}}{\sigma} \\ &= \frac{354 - 355}{1.5} \\ &= \frac{-1}{1.5} \\ &= -\frac{2}{3} \end{aligned}$$

If $x = 357$, then

$$\begin{aligned} z &= \frac{x - \bar{x}}{\sigma} \\ &= \frac{357 - 355}{1.5} \\ &= \frac{2}{1.5} \\ &= \frac{4}{3} \end{aligned}$$

If $x = 358$, then

$$\begin{aligned} z &= \frac{x - \bar{x}}{\sigma} \\ &= \frac{358 - 355}{1.5} \\ &= \frac{3}{1.5} \\ &= 2 \end{aligned}$$

If $x = 352$, then

$$\begin{aligned} z &= \frac{x - \bar{x}}{\sigma} \\ &= \frac{352 - 355}{1.5} \\ &= \frac{-3}{1.5} \\ &= -2 \end{aligned}$$

b. $\bar{x} = 355$ and $\sigma = 1.5$

If $z = 3$, then

$$z = \frac{x - \bar{x}}{\sigma}$$
$$3 = \frac{x - 355}{1.5}$$
$$4.5 = x - 355$$
$$359.5 = x$$
$$x = 359.5 \text{ mL}$$

If $z = -3$, then

$$z = \frac{x - \bar{x}}{\sigma}$$
$$-3 = \frac{x - 355}{1.5}$$
$$-4.5 = x - 355$$
$$350.5 = x$$
$$x = 350.5 \text{ mL}$$

b. Textbook questions 5, 7, and 11 of "Applications and Problem Solving," p. 413

5. a. To make the standard deviation as large as possible, the numbers would be

1, 1, 1, 1, 1, 9, 9, 9, 9, 9

b. To make the standard deviation as large as possible, the numbers would be

1, 1, 1, 1, 5, 9, 9, 9, 9, 9 or 1, 1, 1, 1, 1, 5, 9, 9, 9, 9

7. a. $\bar{x} = 176$ and $\sigma = 8$

If $x = 158$, then

$$z = \frac{x - \bar{x}}{\sigma}$$
$$= \frac{158 - 176}{8}$$
$$= \frac{-18}{8}$$
$$= -2.25$$

If $x = 194$, then

$$z = \frac{x - \bar{x}}{\sigma}$$
$$= \frac{194 - 176}{8}$$
$$= \frac{18}{8}$$
$$= 2.25$$

The z-score range for the allowable heights were $-2.25 \leq z \leq 2.25$.

Section 2: Activity 2 (continued)

b. $\bar{x} = 176$ and $\sigma = 8$, and $z = -1.5$

$$\therefore z = \frac{x - \bar{x}}{\sigma}$$
$$-1.5 = \frac{x - 176}{8}$$
$$-12 = x - 176$$
$$164 = x$$
$$x = 164$$

The applicant's height was 164 cm.

c. $\bar{x} = 163$ and $\sigma = 7$

If $x = 152$, then

$$z = \frac{x - \bar{x}}{\sigma}$$
$$= \frac{152 - 163}{7}$$
$$= \frac{-11}{7}$$
$$\doteq -1.57$$

If $x = 194$, then

$$z = \frac{x - \bar{x}}{\sigma}$$
$$= \frac{194 - 163}{7}$$
$$= \frac{31}{7}$$
$$\doteq 4.43$$

The z-score range for the allowable heights was approximately $-1.57 \leq z \leq 4.43$.

d. $\bar{x} = 163$ and $\sigma = 7$, and $x = 176$

$$\therefore z = \frac{x - \bar{x}}{\sigma}$$
$$= \frac{176 - 163}{7}$$
$$= \frac{13}{7}$$
$$\doteq 1.86$$

The applicant's z-score was about 1.86.

e. The smallest z-score range shows which height requirement was more restrictive.

The z-score range for males was $2.25 + 2.25 = 4.5$. The z-score range for females was $1.57 + 4.43 \doteq 6$.

Therefore, the male height requirement was more restrictive.

11. a. The standard deviation is like a distance: by definition, it is positive.

The standard deviation can be zero if all the data points are the same.

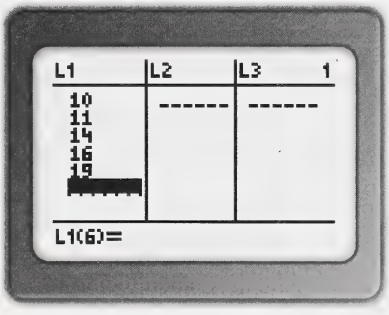
b. The standard deviation can be larger than the mean. The range of numbers has to be large for this to occur. One such sample is $\{0, 0, 1, 15, 20\}$, where $\bar{x} = 7.2$ and $\sigma \doteq 8.6$.

c. If 5 were added to each number in a set of data, the mean would increase by 5 and the standard deviation would remain the same.

If each element in the set of data is multiplied by -3, the mean would be multiplied by -3 and the standard deviation would be multiplied by 3.

4. Textbook question 1 of Investigation 1, "Relating a Normal Curve to the Standard Normal Curve," p. 414

1. a.



The mean is 14, and the standard deviation is approximately 3.286.

b.

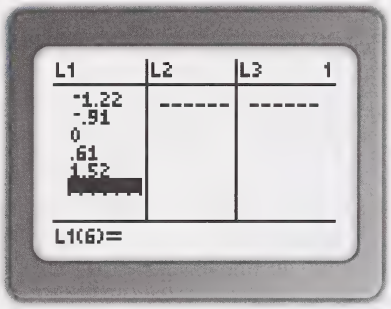
$$z = \frac{x - \bar{x}}{\sigma} \qquad z = \frac{x - \bar{x}}{\sigma} \qquad z = \frac{x - \bar{x}}{\sigma} \qquad z = \frac{x - \bar{x}}{\sigma} \qquad z_{19} = \frac{x - \bar{x}}{\sigma}$$

$$z_{10} \doteq \frac{10 - 14}{3.29} \qquad z_{11} \doteq \frac{11 - 14}{3.29} \qquad z_{14} \doteq \frac{14 - 14}{3.29} \qquad z_{16} \doteq \frac{16 - 14}{3.29} \qquad \doteq \frac{19 - 14}{3.29}$$

$$\doteq -1.217 \qquad \doteq -0.913 \qquad = 0 \qquad \doteq 0.609 \qquad \doteq 1.521$$

Section 2: Activity 2 (continued)

c. Enter the data from the answer to question 1.b.

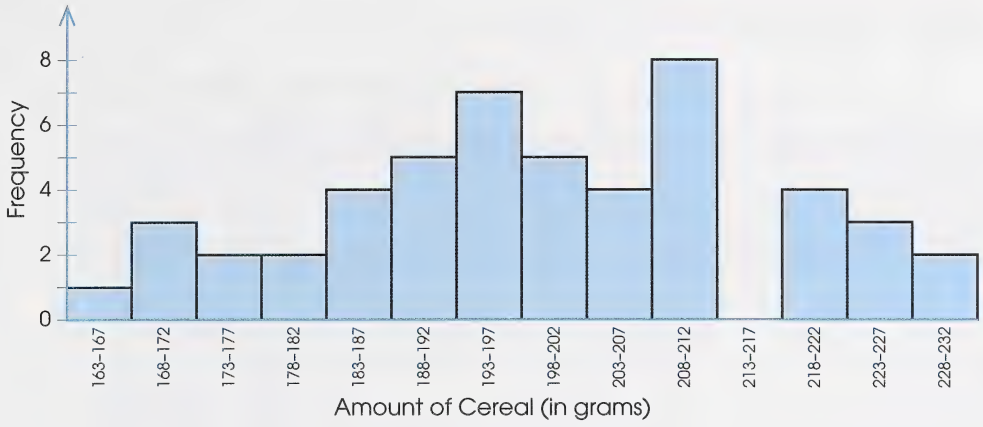


The mean of the z-scores is 0, and the standard deviation is approximately 1.

Section 2: Activity 3

1. a. Textbook questions a. and b. of "Explore: Draw a Graph," p. 418

Range (in grams)	Tally	Range (in grams)	Tally
163–167	𝍸	198–202	𝍸
168–172	𝍸 | | |	203–207	𝍸 |
173–177	𝍸 | |	208–212	𝍸 𝍸 | | |
178–182	𝍸 | |	213–217	
183–187	𝍸 |	218–222	𝍸 |
188–192	𝍸	223–227	𝍸 | | |
193–197	𝍸 | |	228–232	𝍸 | |



b. A manager would probably not accept this day's packaging because too many boxes were under the advertised weight. Customers would have a legitimate reason to complain, thus affecting sales as a result.

b. Textbook questons 1 to 5 of "Inquire," p. 418

1. The mean, $\bar{x}$, is approximately 200 g. The mean is actually 199.28 g.

2. There is more data below the mean than above the mean. (There are 28 data points below the mean and 22 data points above the mean.)

3. As the mass moves away from the mean, the probability of obtaining a package with that mass decreases.

4. The range of masses is as follows:

$$\bar{x} - 16.2 < x < \bar{x} + 16.2$$
$$199.28 - 16.2 < x < 199.28 + 16.2$$
$$183.08 < x < 215.48$$

There are 35 boxes that lie within this range; or 70% of the boxes lie within one standard deviation of the mean.

The range of masses two standard deviations from the mean is as follows:

$$\bar{x} - 2(16.2) < x < \bar{x} + 2(16.2)$$
$$199.28 - 32.4 < x < 199.28 + 32.4$$
$$166.88 < x < 231.68$$

There are 49 boxes that lie within this range; or 98% of the boxes lie within two standard deviations of the mean.

Section 2: Activity 3 (continued)

5. $P(\text{box has more than 225 g}) = \dfrac{\text{number of boxes that contain more than 225 g}}{\text{number of boxes}}$

$$= \frac{4}{50}$$
$$= 0.08$$

The probability of a box containing more than 225 g is 8%.

$$P(\text{box has less than 170 g}) = \frac{\text{number of boxes that contain less than 170 g}}{\text{number of boxes}}$$
$$= \frac{1}{50}$$
$$= 0.02$$

The probability of a box containing less than 170 g is 2%.

2. **Textbook questions 1.a., 1.b., 1.c., 2.a., 2.b., 2.d., 2.e., 3.a., 3.b., 3.f., and 4 of Investigation 2, "The Area Under the Standard Normal Curve," p. 415**

1. a.

	0.00	0.01	0.02	0.03
0.0	0.5000	0.5040	0.5080	0.5120
0.1	0.5398	0.5438	0.5478	0.5517
0.2	0.5793	0.5832	0.5871	0.5910
0.9	0.8159	0.8186	0.8212	0.8238
1.0	0.8413	0.8438	0.8461	0.8485
1.1	0.8643	0.8665	0.8686	0.8708

The area under the curve and to the left of a z-score of 1.13 is 0.8708 or 87.08%.

b. The area under the curve and to the left of a z-score of -2.16 is 0.0154 or 1.54%.

c. The area under the curve and to the left of a z-score of 1.90 is 0.9713 or 97.13%.

2. a. Because the area under the standard normal curve is less than 0.5, the z-score will be negative.

Look for 0.0089 in the main part of the table.

	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07
−2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015
−2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021
−2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028
−2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038
−2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051
−2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068
−2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089

The z-score is -2.37.

b. Because the area under the standard normal curve is less than 0.5, the z-score will be negative.

Look for 0.3409 in the main part of the table. The z-score is -0.41.

d. Because the area under the standard normal curve is greater than 0.5, the z-score will be positive.

Look for 0.9911 in the main part of the table. The z-score is 2.37.

e. Because the area under the standard normal curve is greater than 0.5, the z-score will be positive.

Look for 0.7580 in the main part of the table. The z-score is 0.70.

3. a. $P(z < 1.12) = 0.8686$ and $P(z < 1.98) = 0.9761$

$$\therefore P(1.12 < z < 1.98) = 0.9761 - 0.8686$$
$$= 0.1075$$

Section 2: Activity 3 (continued)

b. $P(z < -2.12) = 0.0170$ and $P(z < -1.0) = 0.1587$

$$\therefore P(-2.12 < z < -1.0) = 0.1587 - 0.0170$$
$$= 0.1417$$

The area between z-scores -2.12 and -1.0 is 0.1417 or 14.17%.

f. $P(z < -1) = 0.1587$ and $P(z < 1) = 0.8413$

$$\therefore P(-1 < z < 1) = 0.8413 - 0.1587$$
$$= 0.6826$$

The area between z-scores -1 and 1 is 0.6826 or 68.26%.

4. a. The area to the left of z-score -2.00 is 0.0228, and the area to the left of z-score 2.00 is 0.9772.

The areas of z-scores -2.00 and 2.00 sum to 1.

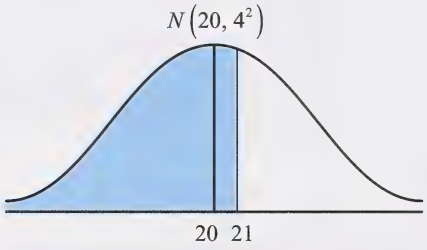
$$0.0028 + 0.9772 = 1.0000$$

b. The area to the left of z-score 0 is 0.5.

c. The area to the left of z-score -0.32 equals the area to the right of z-score 0.32. Therefore, the area to the right of z-score 0.32 is 0.3745.

3. Textbook questions 1, 2, 4, and 6 of "Practice," p. 422

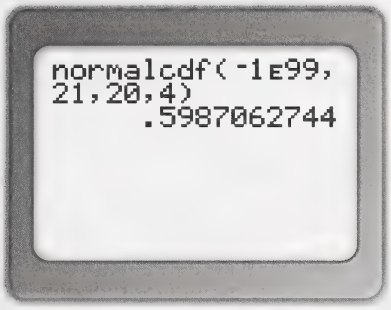
1. Sketch the distribution.



Method 1: Using a Graphing Calculator

$a = -1\text{E}99$, $b = 21$, $\bar{x} = 20$, and $\sigma = 4$

2nd [DISTR] 2 (-) 1 2nd [EE] 9 9 , 2 1
, 2 0 , 4) ENTER



Note:
a = lower bound
b = upper bound
$\bar{x}$ = mean
σ = standard deviation

The percent of data less than 21 g is approximately 59.87 %.

Method 2: Using Tables

First, determine the z-score for 21.

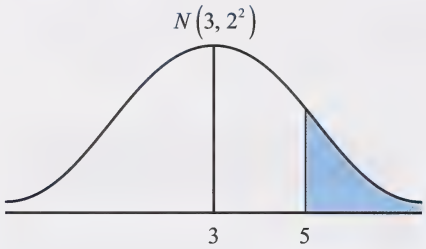
$$z = \frac{x - \bar{x}}{\sigma}$$
$$= \frac{21 - 20}{4}$$
$$= 0.25$$

From the tables, $P(z < 0.25) = 0.5987$.

Therefore, the percent of data less than 21 g is 59.87%.

Section 2: Activity 3 (continued)

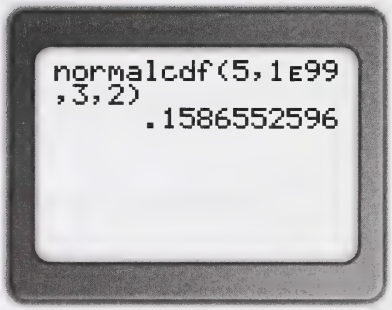
2. Sketch the distribution.



Method 1: Using a Graphing Calculator

$a = 5$, $b = 1\text{E}99$, $\bar{x} = 3$, and $\sigma = 2$

2nd [DISTR] 2 5 , 1 2nd [EE]
9 9 , 3 , 2) ENTER



The percent of data greater than 5 min is approximately 15.87%.

Method 2: Using Tables

First, determine the z-score for 5.

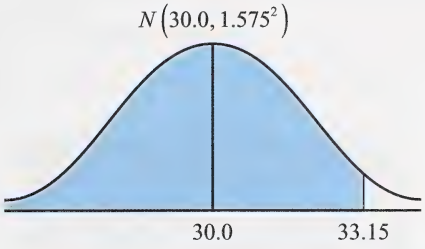
$$z = \frac{x - \bar{x}}{\sigma}$$
$$= \frac{5 - 3}{2}$$
$$= 1$$

From the tables, $P(z < 1) = 0.8413$.

$$\therefore P(z > 1) = 1 - 0.8413$$
$$= 0.1587$$

Therefore, the percent of data greater than 5 min is 15.87%.

4. Sketch the distribution.



Method 1: Using a Graphing Calculator

$a = -1\text{E}99$, $b = 33.15$, $\bar{x} = 30.0$, and $\sigma = 1.575$

2nd [DISTR] 2 (-) 1 2nd [EE] 9
9 , 3 3 • 1 5 , 3 0
, 1 • 5 7 5) ENTER

The percent of data less than 33.15 m is approximately 97.72%.

Method 2: Using Tables

First, determine the z-score for 33.15.

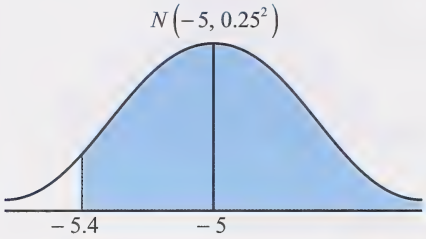
$$z = \frac{x - \bar{x}}{\sigma}$$
$$= \frac{33.15 - 30.0}{1.575}$$
$$= 2$$

From the tables, $P(z < 2) = 0.9772$.

Therefore, the percent of data less than 33.15 m is 97.72%.

Section 2: Activity 3 (continued)

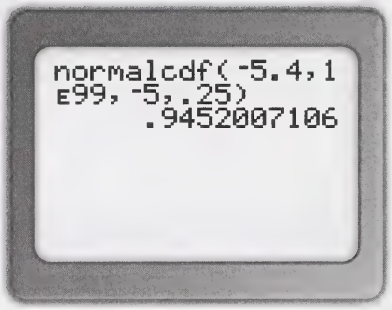
6. Sketch the distribution.



Method 1: Using a Graphing Calculator

$a = -5.4$, $b = 1\text{E}99$, $\bar{x} = -5$, and $\sigma = 0.25$

2nd [DISTR] 2 (–) 5 • 4 ,
1 2nd [EE] 9 9 , (–) 5 ,
• 2 5) ENTER



The percent of data greater than -5.4 is approximately 94.52%.

Method 2: Using Tables

First, determine the z-score for -5.4.

$$z = \frac{x - \bar{x}}{\sigma}$$
$$= \frac{-5.4 - (-5)}{0.25}$$
$$= -1.6$$

From the tables, $P(z < -1.6) = 0.0548$.

$$\therefore P(z > -1.6) = 1 - 0.0548$$
$$= 0.9452$$

Therefore, the percent of data greater than -5.4 is 94.52%.

4. a. Textbook questions 7, 9, 10, 13, 14, and 16 of "Practice," pp. 422 and 423

7. Method 1: Using a Graphing Calculator

$a = 10$, $b = 11$, $\bar{x} = 9$, and $\sigma = 2$

The percent of data that lies between 10 cm and 11 cm is approximately 14.99%.

Method 2: Using Tables

First, determine the z-scores for 10 and 11.

$$z = \frac{x - \bar{x}}{\sigma}$$
$$z_{10} = \frac{10 - 9}{2}$$
$$= 0.5$$

$$z = \frac{x - \bar{x}}{\sigma}$$
$$z_{11} = \frac{11 - 9}{2}$$
$$= 1$$

From the tables, $P(z_{10} < 0.5) = 0.6915$ and $P(z_{11} < 1)$ 0.8413.

$$\therefore P(0.5 < z < 1) = 0.8413 - 0.6915$$
$$= 0.1498$$

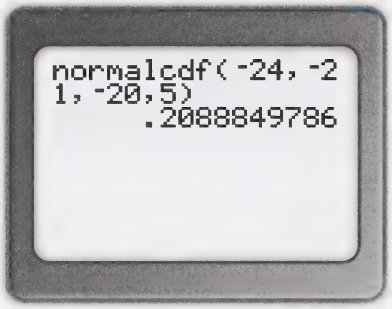
The percent of data that lies between 10 cm and 11 cm is 14.98%.

Note: Differences between answers arise because of rounding.

Section 2: Activity 3 (continued)

9. Method 1: Using a Graphing Calculator

$a = -24$, $b = -21$, $\bar{x} = -20$, and $\sigma = 5$



The percent of data that lies between –24°C and –21°C is approximately 20.89%.

Method 2: Using Tables

First, determine the z-scores of -24 and -21.

$$z = \frac{x - \bar{x}}{\sigma}$$

$$z_{-24} = \frac{-24 - (-20)}{5}$$

$$= -0.8$$

$$z = \frac{x - \bar{x}}{\sigma}$$

$$z_{-21} = \frac{-21 - (-20)}{5}$$

$$= -0.2$$

From the tables, $P(z_{-24} < -0.8) = 0.2119$ and $P(z_{-21} < -0.2) = 0.4207$.

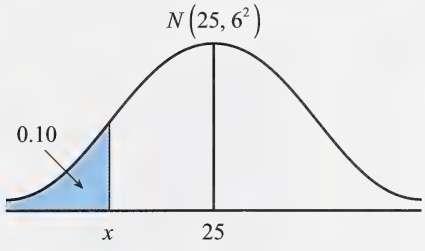
$$\therefore P(-0.8 < z < -0.2) = 0.4207 - 0.2119$$

$$= 0.2088$$

Note: Differences between answers arise because of rounding.

The percent of data that lies between –24°C and –21°C is 20.88%.

10. Sketch the distribution.



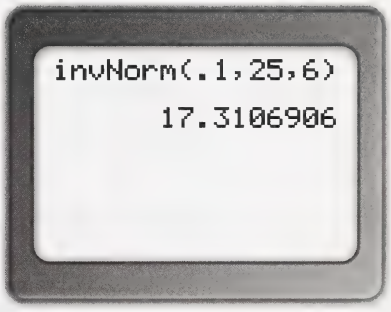
The value obtained from the calculator is the x-value associated with the area to the left of the x-value.

Method 1: Using a Graphing Calculator

Using a graphing calculator, the inverse function is required because an area is known.

area = 0.10, $\bar{x} = 25$, and $\sigma = 6$

2nd [DISTR] 3 (invNorm() • 1 , 2 5 , 6) ENTER



The measurement that satisfies the given facts is approximately 17.31 h.

Method 2: Using Tables

Using the tables, look for an area closest to 0.1000.

The corresponding z-score for an area of 0.1000 is approximately -1.28.

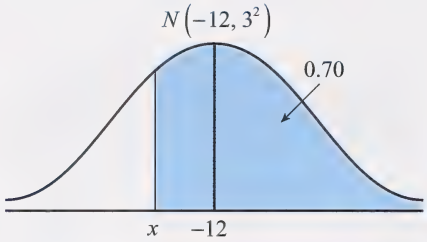
Solve for x.

$$z = \frac{x - \bar{x}}{\sigma}$$
$$-1.28 \doteq \frac{x - 25}{6}$$
$$x \doteq -1.28(6) + 25$$
$$\doteq 17.32$$

The measurement that satisfies the given facts is approximately 17.32 h.

Section 2: Activity 3 (continued)

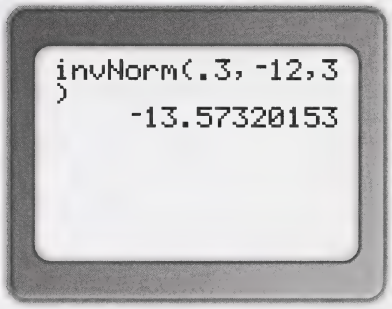
13. Sketch the distribution.



Method 1: Using a Graphing Calculator

The area to the left is $1.0 - 0.7 = 0.3$.

$\therefore$ area $= 0.3$, $\bar{x} = -12$, and $\sigma = 3$.



2nd [DISTR] 3 • 3 , (-) 1

2 , 3) ENTER

The measurement that satisfies the given facts is approximately $-13.57°C$.

Method 2: Using Tables

Using the tables, look for an area closest to 0.3000.

The corresponding z-score for an area of 0.3000 is approximately -0.52.

Solve for x.

$$z = \frac{x - \bar{x}}{\sigma}$$

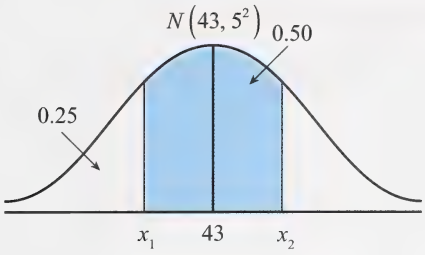
$$-0.52 \doteq \frac{x - (-12)}{3}$$

$$x \doteq -0.52(3) - 12$$

$$\doteq -13.56$$

Therefore, the measurement that satisfies the given facts is approximately $-13.56°C$.

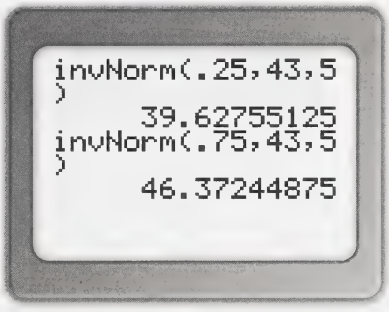
14. Sketch the distribution.



Method 1: Using a Graphing Calculator

The area to the left of the lower bound is 0.25, and the area to the left of the upper bound is 0.75.

$\therefore$ area$_1$ = 0.25, $\bar{x} = 43$, $\sigma = 5$, and area$_2$ = 0.75



The range is approximately 39.63 years to 46.37 years.

Method 2: Using Tables

Using the tables, look for areas closest to 0.2500 and 0.7500.

The corresponding z-score for an area of 0.2500 is approximately -0.67, and the corresponding z-score for an area of 0.7500 is approximately 0.67.

Section 2: Activity 3 (continued)

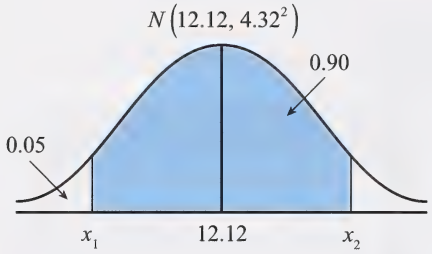
Use the z-scores to determine the interval.

$$z = \frac{x - \bar{x}}{\sigma}$$
$$-0.67 \doteq \frac{x_1 - 43}{5}$$
$$x_1 \doteq -0.67(5) + 43$$
$$\doteq 39.65$$

$$z = \frac{x - \bar{x}}{\sigma}$$
$$0.67 \doteq \frac{x_2 - 43}{5}$$
$$x_2 \doteq 0.67(5) + 43$$
$$\doteq 46.35$$

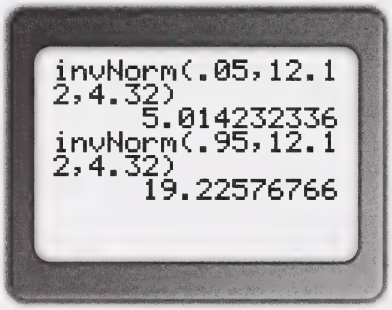
The range is approximately 39.65 years to 46.35 years.

16. Sketch the distribution.



Method 1: Using a Graphing Calculator

area$_1$ = 0.05, $\bar{x}$ = 12.12, σ = 4.32, and area$_2$ = 0.95



The range is \$5.01 to \$19.23.

Method 2: Using Tables

Using the tables, look for areas closest to 0.0500 and 0.9500.

The corresponding z-score for an area of 0.0500 is -1.645 (the average of -1.64 and -1.65), and the corresponding z-score for an area of 0.9500 is approximately 1.645 (the average of 1.64 and 1.65).

Use the z-scores to determine the range.

$$z = \frac{x - \bar{x}}{\sigma}$$

$$-1.645 \doteq \frac{x_1 - 12.12}{4.32}$$

$$x_1 \doteq -1.645(4.32) + 12.12$$

$$\doteq 5.0136$$

$$z = \frac{x - \bar{x}}{\sigma}$$

$$1.645 \doteq \frac{x_2 - 12.12}{4.32}$$

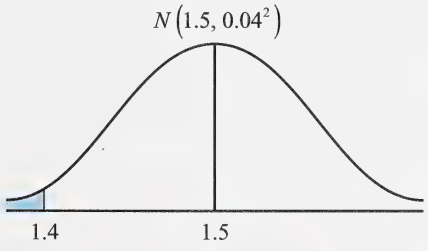
$$x_2 \doteq -1.645(4.32) + 12.12$$

$$\doteq 19.2264$$

The range is \$5.01 to \$19.23.

b. Textbook questions 18, 20, 21, 22, and 24.a. of "Applications and Problem Solving," p. 423

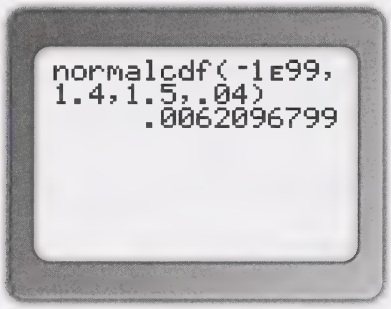
18. a. Sketch the distribution.



Section 2: Activity 3 (continued)

Method 1: Using a Graphing Calculator

$a = -1\text{E}99$, $b = 1.4$, $\bar{x} = 1.5$, and $\sigma = 0.04$



Approximately 0.62% of the strands will have a strength less than 1.4 N.

Method 2: Using Tables

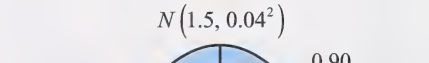
Find the z-score for 1.4.

$$z = \frac{x - \bar{x}}{\sigma}$$
$$= \frac{1.4 - 1.5}{0.04}$$
$$= -2.5$$

From the tables, $P(z < -2.5) = 0.0062$.

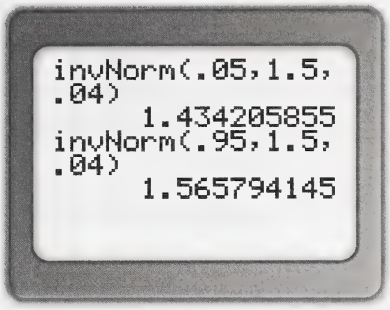
Therefore, approximately 0.62% of the strands will have a strength less than 1.4 N.

b. Sketch the distribution.



Method 1: Using a Graphing Calculator

$\text{area}_1 = 0.05$, $\bar{x} = 1.5$, $\sigma = 0.04$, and $\text{area}_2 = 0.95$



The range of strengths is approximately 1.43 N to 1.57 N.

Method 2: Using Tables

Using the tables, look for areas closest to 0.0500 and 0.9500.

The corresponding z-score for an area of 0.0500 is approximately -1.645 (the average of -1.64 and -1.65) and the corresponding z-score for an area of 0.9500 is approximately 1.645 (the average of 1.64 and 1.65).

Use the z-scores to determine the range.

$$z = \frac{x - \bar{x}}{\sigma} \qquad z = \frac{x - \bar{x}}{\sigma}$$

$$-1.645 \doteq \frac{x_1 - 1.5}{0.04} \qquad 1.645 \doteq \frac{x_2 - 1.5}{0.04}$$

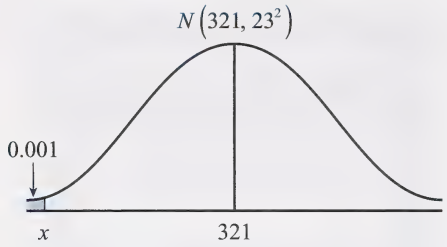
$$-0.0658 \doteq x_1 - 1.5 \qquad 0.0658 \doteq x_2 - 1.5$$

$$x_1 \doteq 1.4342 \qquad x_2 \doteq 1.5658$$

The range of strengths is approximately 1.43 N to 1.57 N.

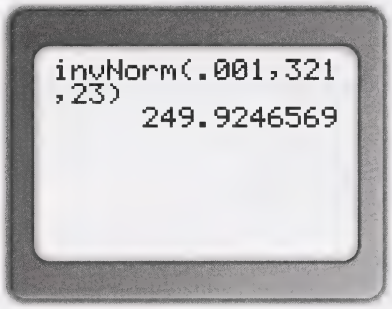
Section 2: Activity 3 (continued)

20. Sketch the distribution.



Method 1: Using a Graphing Calculator

area = 0.001, $\bar{x} = 321$, and $\sigma = 23$



The component should be replaced after about 250 h of use.

Method 2: Using Tables

Using the tables, look for an area close to 0.0010.

The corresponding z-score for an area of 0.0010 is approximately -2.99.

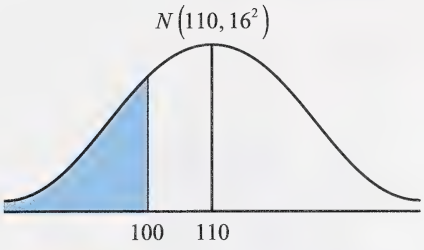
Solve for x.

$$z = \frac{x - \bar{x}}{\sigma}$$
$$-2.99 = \frac{x - 321}{23}$$
$$-68.77 = x - 321$$
$$x = 252.23$$

Note: The answers to Method 1 and Method 2 differ slightly due to rounding.

The component should be replaced after about 252 h of use.

21. a. Sketch the distribution.



Method 1: Using a Graphing Calculator

$a = -1\text{E}99$, $b = 100$, $\bar{x} = 110$, and $\sigma = 16$

About 26.6% of motorists drive less than or at the posted speed limit.

Method 2: Using Tables

Find the z-score for 100.

$$z = \frac{x - \bar{x}}{\sigma}$$
$$= \frac{100 - 110}{16}$$
$$= -0.625$$

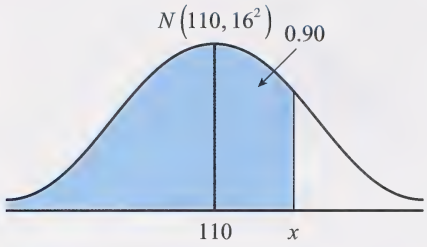
Note: The answers to Method 1 and Method 2 differ slightly due to rounding.

From the tables, $P(z < -0.625) \doteq 0.2643$.

Therefore, about 26.4% of motorists drive less than or at the posted speed limit.

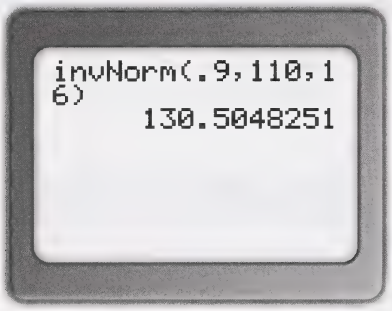
Section 2: Activity 3 (continued)

b. Sketch the distribution.



Method 1: Using a Graphing Calculator

area = 0.90, $\bar{x} = 110$, and $\sigma = 16$



Therefore, 90% of motorists drive below 130.5 km/h.

Method 2: Using Tables

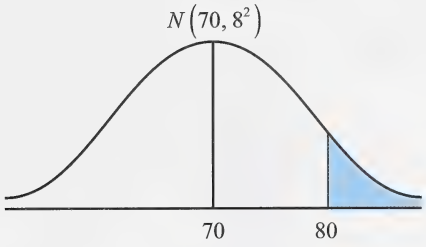
The z-score for an area of 0.9000 is approximately 1.28.

Solve for x.

$$z = \frac{x - \bar{x}}{\sigma}$$
$$1.28 \doteq \frac{x - 110}{16}$$
$$20.48 \doteq x - 110$$
$$x \doteq 130.48$$

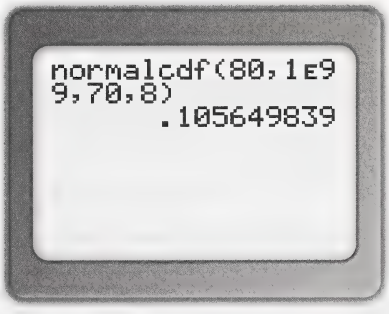
Therefore, 90% of motorists drive below 130.5 km/h.

22. a. Sketch the distribution.



Method 1: Using a Graphing Calculator

$a = 80$, $b = 1\text{E}99$, $\bar{x} = 70$, and $\sigma = 8$



Therefore, about 10.6% of the students obtained a mark above 80.

Method 2: Using Tables

Find the z-score for 80.

$$z = \frac{x - \bar{x}}{\sigma}$$
$$= \frac{80 - 70}{8}$$
$$= 1.25$$

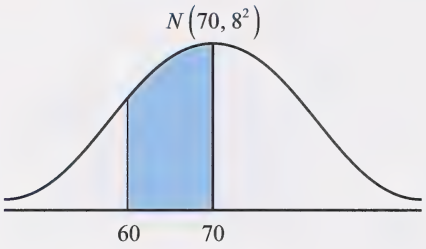
From the tables, $P(z < 1.25) = 0.8944$.

$$\therefore P(z > 1.25) = 1 - 0.8944$$
$$= 0.1056$$

Therefore, 10.6% of the students obtained a mark above 80.

Section 2: Activity 3 (continued)

b. Sketch the distribution.



Method 1: Using a Graphing Calculator

$a = 60$, $b = 70$, $\bar{x} = 70$, and $\sigma = 8$

Therefore, about 39.4% of the students obtained a C grade.

Method 2: Using Tables

Because 70 is the mean, the area to the left of 70 is 0.5.

To calculate the area to the left of 60, determine the z-score.

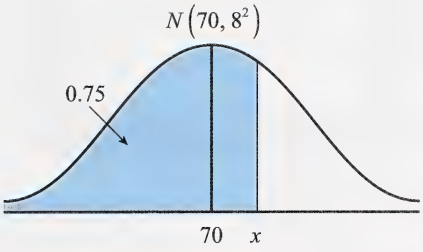
$$z = \frac{x - \bar{x}}{\sigma}$$
$$= \frac{60 - 70}{8}$$
$$= -1.25$$

From the tables, $P(z < -1.25) = 0.1056$.

$$\therefore P(-1.25 < z < 0.00) = 0.5000 - 0.1056$$
$$= 0.3944$$

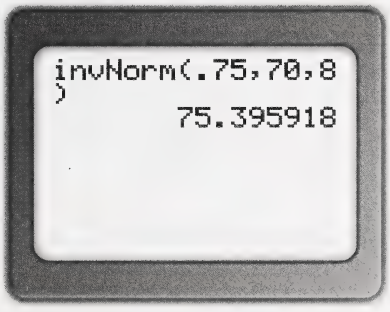
Therefore, 39.4% of the students obtained a C grade.

c. Sketch the distribution.



Method 1: Using a Graphing Calculator

area = 0.75, $\bar{x} = 70$, and $\sigma = 8$



Therefore, 75% of the students obtained a mark less than 75.4.

Section 2: Activity 3 (continued)

Method 2: Using Tables

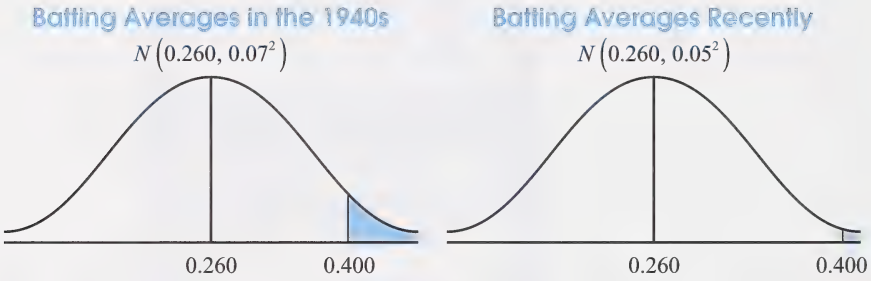
The z-score for an area of 0.7500 is about 0.67.

Solve for x.

$$z = \frac{x - \bar{x}}{\sigma}$$
$$0.67 \doteq \frac{x - 70}{8}$$
$$5.36 \doteq x - 70$$
$$x \doteq 75.36$$

Therefore, 75% of the students obtained a mark less than 75.4.

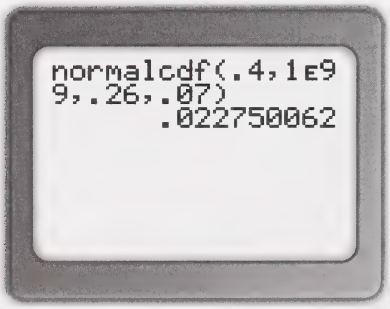
24. a. Sketch the distributions.



Method 1: Using a Graphing Calculator

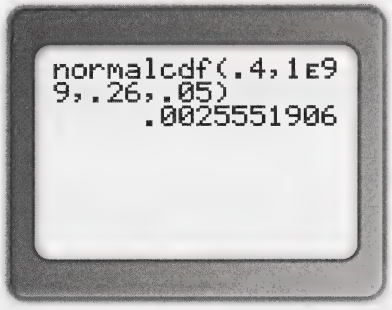
In the 1940s

$a = 0.400$, $b = 1\text{E}99$, $\bar{x} = 0.260$, and $\sigma = 0.07$



Recently

$a = 0.400$, $b = 1\text{E}99$, $\bar{x} = 0.260$, and $\sigma = 0.05$



In the 1940s, about 2.28% of the players batted over 400. Recently, about 0.26% of the players batted over 400.

Method 2: Using Tables

In the 1940s

Determine the z-score for 0.400.

$$z = \frac{x - \bar{x}}{\sigma}$$
$$= \frac{0.400 - 0.260}{0.07}$$
$$= 2$$

From the tables, $P(z < 2) = 0.9772$.

$$\therefore P(z > 2) = 1 - 0.9772$$
$$= 0.0228$$

Recently

Determine the z-score for 0.400.

$$z = \frac{x - \bar{x}}{\sigma}$$
$$= \frac{0.400 - 0.260}{0.05}$$
$$= 2.8$$

From the tables, $P(z < 2.8) = 0.9974$.

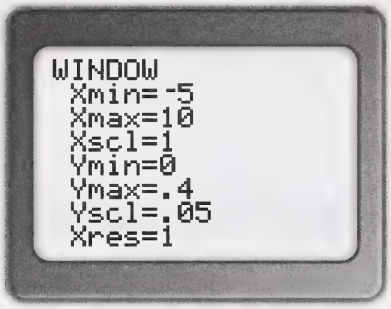
$$\therefore P(z > 2.8) = 1 - 0.9974$$
$$= 0.0026$$

In the 1940s, about 2.28% of the players batted over 400. Recently, about 0.26% of the players batted over 400.

Section 2: Activity 3 (continued)

5. Textbook question 25 of "Applications and Problem Solving," p. 423

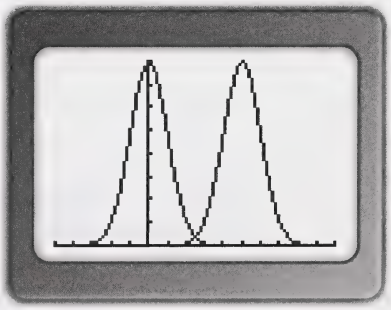
25. a. Use the following window settings.



For the graph of $N(0, 1)$, $\bar{x} = 0$ and $\sigma^2 = 1$; and for the graph of $N(5, 1)$, $\bar{x} = 5$ and $\sigma^2 = 1$.

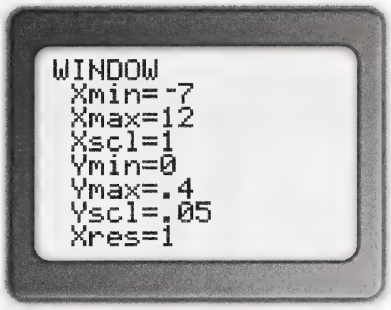
Y= (1 ÷ 2nd [√] 2 2nd [π])) × 2nd [e^x] (–)
(X,T,Θ,*n* – 0) x^2 ÷ (2 × 1)) ENTER

(1 ÷ 2nd [√] 2 2nd [π])) × 2nd [e^x] (–) (
X,T,Θ,*n* – 5) x^2 ÷ (2 × 1)) GRAPH

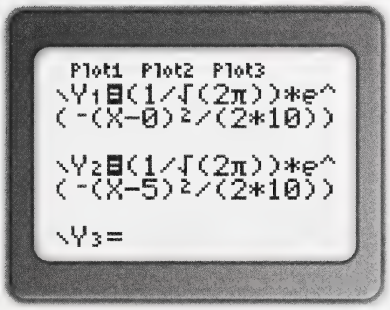


The graph of $N(5, 1)$ is the graph of $N(0, 1)$ translated 5 units to the right.

b. Use the following window settings.



For the graph of $N(0, 10)$, $\bar{x} = 0$ and $\sigma^2 = 10$; and for the graph of $N(5, 10)$, $\bar{x} = 5$ and $\sigma^2 = 10$.

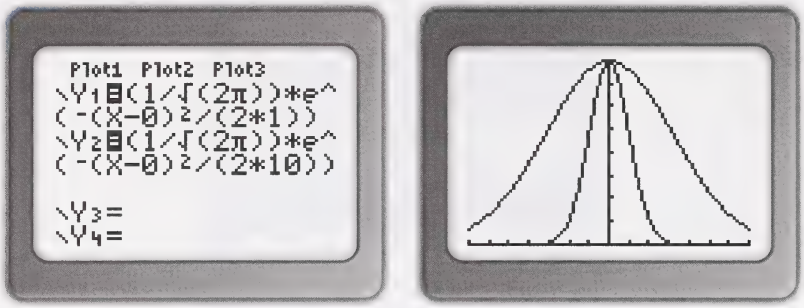


The graph of $N(5, 10)$ is the graph of $N(0, 10)$ translated 5 units to the right.

c. Use the following window settings.

Section 2: Activity 3 (continued)

For the graph of $N(0, 1)$, $\bar{x} = 0$ and $\sigma^2 = 1$; and for the graph of $N(0, 10)$, $\bar{x} = 0$ and $\sigma^2 = 10$.



The graph of $N(0, 10)$ is the graph of $N(0, 1)$ stretched horizontally by a factor of 10.

Section 2: Follow-up Activities

Extra Help

1. a. $\sigma = \sqrt{npq}$

b. $\bar{x} = np$

c. $\sigma = \sqrt{\dfrac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{n}}$

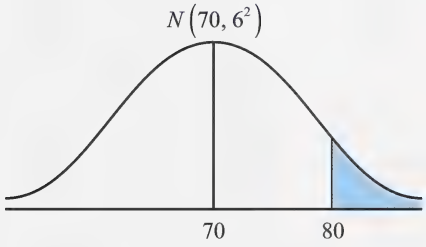
2. Textbook questions 5, 10, and 11.a. of "Chapter Check," p. 442

5. a. To determine the probability of given masses, the z-scores need to be determined.

Greater than 80 kg

$$z = \frac{x - \bar{x}}{\sigma}$$
$$= \frac{80 - 70}{6}$$
$$\doteq 1.67$$

Sketch the distribution.

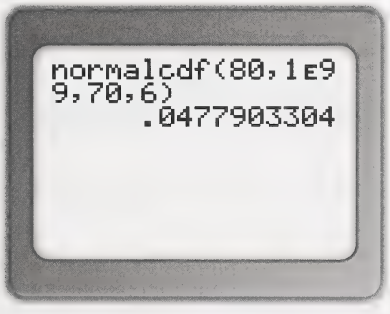


Using the tables, the area to the left of $z = 1.67$ is 0.9525. To calculate the area to the right, subtract 0.9525 from 1.

$$P(z > 1.67) = 1 - 0.9525$$
$$= 0.0475$$

Check

$a = 0.80$, $b = 1\text{E}99$, $\bar{x} = 70$, and $\sigma = 6$



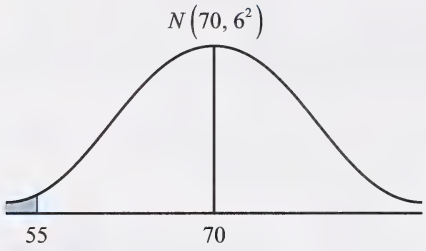
The probability that the student has a mass of more than 80 kg is 0.048 or 4.8%.

Extra Help (continued)

Less than 55 kg

$$z = \frac{x - \bar{x}}{\sigma}$$
$$= \frac{55 - 70}{6}$$
$$\doteq -2.5$$

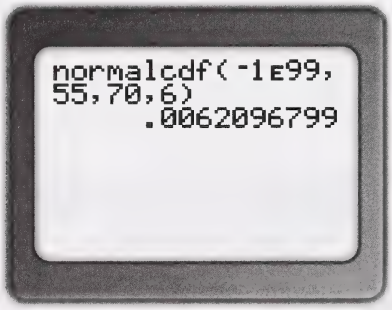
Sketch the distribution.



Using the tables, the area to the left of $z = -2.5$ is 0.0062.

Check

$a = -1\text{E}99$, $b = 55$, $\bar{x} = 70$, and $\sigma = 6$



The probability that the student has a mass less than 55 kg is approximately 0.006 or 0.6%.

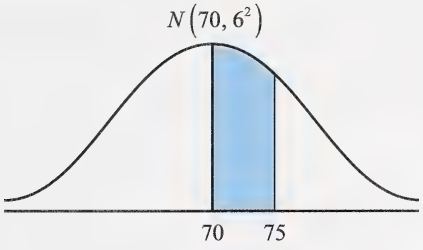
Between 70 kg and 75 kg

Because 70 is the mean, $z_{70} = 0$.

Determine z_{75}.

$$z = \frac{x - \bar{x}}{\sigma}$$
$$z_{75} = \frac{75 - 70}{6}$$
$$\doteq 0.83$$

Sketch the distribution.



The area to the left of the mean is 0.5. The area to the left of 0.83 can be found by using the tables on pages 416 and 417 of the textbook. This area is 0.7967. To determine the area between these two values, subtract 0.5 from 0.7967.

$$\text{Area} = 0.7967 - 0.5000$$
$$= 0.2967$$

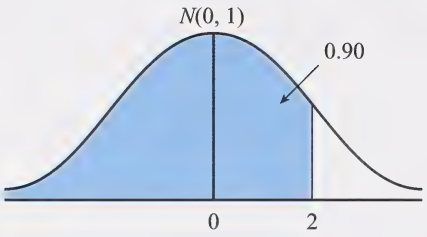
Check

$a = 70$, $b = 75$, $\bar{x} = 70$, and $\sigma = 6$

The probability that the student has a mass between 70 kg and 75 kg is about 0.297 or 29.7%.

Extra Help (continued)

b. Sketch the distribution.



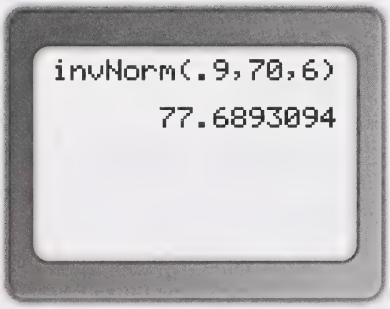
The area to the left of z must be 0.90. To determine the value of z, look for an area closest to 0.9000 in the table. An area of 0.8997 is closest to 0.90; so, the value of z is about 1.28.

Convert the z-score to the corresponding mass.

$$z = \frac{x - \bar{x}}{\sigma}$$
$$1.28 \doteq \frac{x - 70}{6}$$
$$x \doteq (1.28)(6) + 70$$
$$\doteq 77.68$$

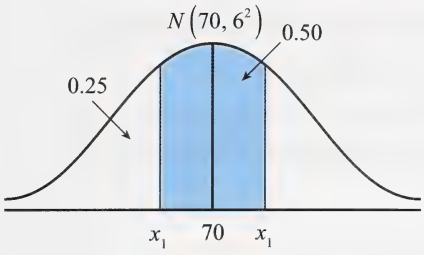
Check

area = 0.90, $\bar{x} = 70$, and $\sigma = 6$



About 90% of the male students have a mass less than 78 kg.

c. Sketch the distribution.



Since the area to the left of x_1 is 0.25, the z-score can be found by finding an area closest to 0.2500 in the table. The area closest to 0.25 is 0.2514, which corresponds to a z-score of -0.67.

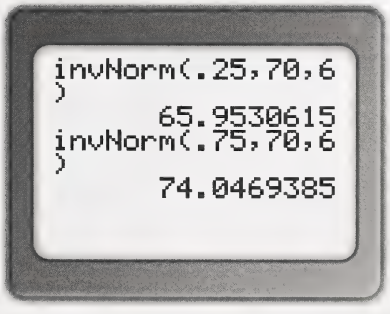
Convert a z-score of -0.67 to the corresponding mass.

$$z = \frac{x - \bar{x}}{\sigma}$$
$$-0.67 \doteq \frac{x_1 - 70}{6}$$
$$x_1 \doteq (-0.67)(6) + 70$$
$$\doteq 65.98$$

The mass is approximately 66 kg. Using symmetry, the value that corresponds to x_2 is $70 + (70 - 66) = 74$ kg.

Check

area = 0.25, $\bar{x} = 70$, and $\sigma = 6$, and $\text{area}_2 = 0.75$



The range of measurements that contains 50% of the data is from 66 kg to 74 kg.

Extra Help (continued)

10. a. Each product can be considered an independent trial. Here, a success is being not defective.

$n = 2000$ (number of trials)
$p = 0.999$ (probability of no defects on each trial)
$q = 0.001$ (probability of defect on each trial)

Since $x = 2000$,

$$\begin{aligned} P(\text{no defects}) &= {}_nC_x p^x q^{n-x} \\ &= {}_{2000}C_{2000}\,(0.999)^{2000}\,(0.001)^0 \\ &\doteq 0.135\,199\,925\,4 \end{aligned}$$

$x = 2000$, because all the items are "good".

The probability that no defective products are made in a day is about 13.52%.

b. The probability that at least one defective product is made is the probability that no defectives products are made subtracted from 1.

$$\begin{aligned} \therefore\ P(\text{at least 1 defective}) &= 1 - P(\text{no defectives}) \\ &= 1 - 0.1352 \\ &= 0.8648 \end{aligned}$$

The probability that at least one defective product is made in a day is 86.48%.

c. $p = 0.001$ (the probability it is defective)
$q = 0.999$ (the probability it is not defective)
$n = 2000$ (the number of trials)

$$\begin{aligned} P\left(\begin{matrix}\text{at least 2} \\ \text{defective products}\end{matrix}\right) &= 1 - P(\text{no defectives}) - P(1 \text{ defective}) \\ &= 1 - \left[{}_{2000}C_0\,(0.001)^0\,(0.999)^{2000}\right] - \left[{}_{2000}C_1\,(0.001)^1\,(0.999)^{1999}\right] \\ &\doteq 1 - 0.1352 - 0.2707 \\ &\doteq 0.5941 \end{aligned}$$

The probability that at least two defective products are made in a day is approximately 59.41%.

11. a. Since 45 of 75 people preferred mesquite to red chile, the proportion of people is $\frac{45}{75}$ or 0.6.

Therefore, 1000(0.6) = 600 people are expected to choose mesquite over red chile.

Enrichment

Textbook questions 1, 2, and 3.a. of "Exploring Math: The Waiting Period," p. 441

1. a. The probability of rolling a 2 with one die is $\frac{1}{6}$.

The probability of rolling a 2 for the first time on the third roll is

$$P(x) = q^x p$$
$$P(2) = \left(\frac{5}{6}\right)^2 \left(\frac{1}{6}\right)$$
$$\doteq 0.1157 \text{ or } 11.57\%$$

b. The probability of rolling a 3 or a 4 is $\frac{2}{6}$ or $\frac{1}{3}$. Since $p = \frac{1}{3}$, then $q = \frac{2}{3}$. Since there are four rolls,

$$P(x) = q^x p$$
$$P(3) = \left(\frac{2}{3}\right)^3 \left(\frac{1}{3}\right)$$
$$\doteq 0.0988$$

The probability of rolling a 3 or a 4 for the first time on the fourth roll is about 9.88%.

2. a. The sample space of rolling a sum of 8 is {(2, 6), (3, 5), (4, 4), (5, 3), (6, 2)}, where the point (*a*, *b*) represents rolling the number *a* on the first die and *b* on the second die.

Since there are one to three rolls, $x = 0$, $x = 1$, and $x = 2$.

$p = \frac{5}{36}$ and $q = \frac{31}{36}$

$$\therefore P(\text{a sum of 8 in 3 rolls}) = P(0) + P(1) + P(2)$$
$$= \left(\frac{31}{36}\right)^0 \left(\frac{5}{36}\right) + \left(\frac{31}{36}\right)\left(\frac{5}{36}\right) + \left(\frac{31}{36}\right)^2 \left(\frac{5}{36}\right)$$
$$\doteq 0.3615$$

The probability of rolling a sum of 8 in three rolls is approximately 36.15%.

Enrichment (continued)

b. The probability of rolling a sum of 8 in fewer than three rolls is equivalent to the probability of rolling a sum of 8 in one or two rolls.

$\therefore\ p = \frac{5}{36}$, $q = \frac{31}{36}$, and $x = 0, 1$

$$\therefore\ P(x<2) = P(0) + P(1)$$
$$= \left(\frac{31}{36}\right)^0\left(\frac{5}{36}\right)^1 + \left(\frac{31}{36}\right)^1\left(\frac{5}{36}\right)$$
$$\doteq 0.2585$$

The probability of rolling a sum of 8 in fewer than three rolls is approximately 25.85%.

c. The answer to question 2.a., $\frac{5}{36} + \frac{31}{36}\left(\frac{5}{36}\right) + \left(\frac{31}{36}\right)^2\frac{5}{36}$, is a geometric series with a common ratio of $\frac{31}{36}$. The answer to question 2.b., $\frac{5}{36} + \left(\frac{31}{36}\right)\left(\frac{5}{36}\right)$, is also a geometric series with a common ratio of $\frac{31}{36}$.

The answer to question 2.a. is the sum of three terms, and the answer to question 2.b. is the sum of two terms.

3. a. **Drawing an Ace**

The probability of drawing an ace from a regular deck is $\frac{4}{52}$ or $\frac{1}{13}$. So, $p = \frac{1}{13}$ and $q = \frac{12}{13}$. There are at most five cuts; so, $x = 0$, 1, 2, 3, or 4.

$$\therefore\ P(\text{ace in 5 cuts}) = \left(\frac{12}{13}\right)^0\left(\frac{1}{13}\right) + \left(\frac{12}{13}\right)^1\left(\frac{1}{13}\right) + \left(\frac{12}{13}\right)^2\left(\frac{1}{13}\right) + \left(\frac{12}{13}\right)^3\left(\frac{1}{13}\right) + \left(\frac{12}{13}\right)^4\left(\frac{1}{13}\right)$$

This sum is a geometric series, where $a = \frac{1}{13}$ and $r = \frac{12}{13}$.

$$\therefore\ P(\text{ace in 5 cuts}) = \frac{\frac{1}{13}\left[1-\left(\frac{12}{13}\right)^5\right]}{1-\frac{12}{13}}$$
$$\doteq 0.3298$$

Recall that the sum of a geometric sequence is

$$S_n = \frac{a(r^n-1)}{r-1} \text{ or } \frac{a(1-r^n)}{1-r}$$

The probability of drawing an ace in five cuts is approximately 32.98%.

Drawing a Face Card

The probability of drawing a face card is $\frac{12}{52}$ or $\frac{3}{13}$.

$$\therefore P(\text{face card in 5 cuts}) = \left(\frac{10}{13}\right)^0\left(\frac{3}{13}\right) + \left(\frac{10}{13}\right)^1\left(\frac{3}{13}\right) + \left(\frac{10}{13}\right)^2\left(\frac{3}{13}\right) + \left(\frac{10}{13}\right)^3\left(\frac{3}{13}\right) + \left(\frac{10}{13}\right)^4\left(\frac{3}{13}\right)$$

Since $a = \frac{3}{13}$ and $r = \frac{10}{13}$,

$$P(\text{face card in 5 cuts}) = \frac{\frac{3}{13}\left[1-\left(\frac{10}{13}\right)^5\right]}{1-\frac{10}{13}}$$
$$\doteq 0.7307$$

The probability of cutting a face card in five cuts is approximately 73.07%.

Drawing an Ace or a Face Card

The probability of an ace or face card is $\frac{16}{52}$ or $\frac{4}{13}$. So, $p = \frac{4}{13}$, $q = \frac{9}{13}$, and $0 \le x \le 4$, where $x \in N$.

$$\therefore P\begin{pmatrix}\text{ace or face card}\\ \text{in 5 cuts}\end{pmatrix} = \left(\frac{9}{13}\right)^0\left(\frac{4}{13}\right)^1 + \left(\frac{9}{13}\right)^1\left(\frac{4}{13}\right) + \left(\frac{9}{13}\right)^2\left(\frac{4}{13}\right) + \left(\frac{9}{13}\right)^3\left(\frac{4}{13}\right) + \left(\frac{9}{13}\right)^4\left(\frac{4}{13}\right)$$
$$= \frac{\frac{4}{13}\left[1-\left(\frac{9}{13}\right)^5\right]}{1-\frac{9}{13}}$$
$$\doteq 0.8410$$

The probability of drawing an ace or face card in five cuts or less is approximately 84.10%.

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